THE ITERATIVE ALGORITHM OF TUNING CONTROLLERS TO THE MODELS OBJECT WITH INERTIA AND NONMINIMAL PHASE

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REZUMAT. În lucrare se propune un algoritm iterativ de acordare a regulatoarelor tipizate P, PI şi PID la modele de obiecte cu inerţie şi fază neminimă. Algoritmul propus utilizează metoda gradului maximal de stabilitate la acordarea regulatoarelor. Se analizează algoritmul iterativ propus de acordare a regulatoarelor şi se determină performanţele sistemului automat în dependenţă de valoarea gradului maximal de stabilitate al sistemului autamat.

Cuvinte cheie: algoritm iterativ, metoda gradului maximal de stabilitate, acordarea regulatoarelor

ABSTRACT. In this paper is proposed the iterative algorithm of tuning the typical controllers P, PI, PID to the model objects with inertia and nonminimal phase. In the proposed algorithm it is using the maximal stability degree method for tuning controllers. In the result of this studding it is proposed the algorithm of tuning controllers and the procedure of determining the system's performance in dependence of maximal stability value.

Keywords: the iterative algorithm, the maximal stability degree method, tuning of controllers.

1. INTRODUCTION

At the automation of many slow technological processes the mathematical objects' models of control process are represented as the models with respectively order inertia [1, 2].

The procedure of tuning controllers to the model object with inertia and nonminimal phase becomes difficult. In this paper is analyzing the model object (fixed part) with inertia (third order) and nonminimal phase with transfer function which is presented in the follow form

$$H_{PF}(s) = \frac{k}{(T_1 s - 1)(T_2 s^2 + T_3 s + 1)} = \frac{k}{a_0 s^3 + a_1 s^2 + a_2 s - a_3}, \quad (1)$$

where the parameters of object k, T_1, T_2, T_3 and $a_0=T_1T_2$, $a_1= T_1T_3 - T_2$, $a_2=T_1$-T_3, $a_3=1$ are known.

To the model object (1) with known parameters k, T_1, T_2, T_3 it is proposed to tune the standard controllers P, PI and PID using the maximal stability degree (M.S.D) method [3] and to analyze the dynamic of control system (CS) for the case when it is varying the object's parameters from the nonminimal values k, T_1, T_2, T_3, keeping the tuning parameters of controllers P, PI and PID.

2. THE TUNING ALGORITHM OF CONTROLLERS

Assume that the control system is formed of object with transfer function $H_{PF}(s)$, which is presented in relation (1), and transfer function of controller $H_R(s)$ with typical control laws P, PI, PID (figure 1).

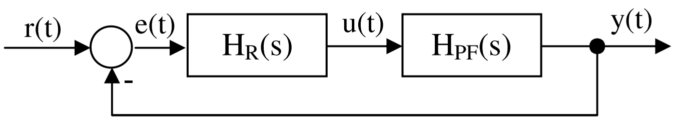


Fig. 1. The structure scheme of control system.

It will be tune the typical algorithms of tuning P, PI and PID for the model object with known parameters, using the GMS method [3, 4, 5].

For the tuning the P controller using the GMS method it was applying the algebraic expressions, which are the analytical expressions

$$-3a_0J^2 + 2a_1J - a_2 = 0 , \quad (2)$$

$$k_p = (1/k)(a_0J^3 - a_1J^2 + a_2J + a_3). \quad (3)$$

The expression (2) is a function of object's parameters and unknown value of stability degree J. Solved the expression (2) it is determined the optimal value of stability degree J_{opt} , which presented the smallest positive and real root or the real positive part of complex root.

From expression (3) were determined the optimal values of parameters k_p of P controller.

For the tuning the PI controller using the GMS method it was applying the algebraic expressions which are the analytical expressions

$$6a_0J^2 - 3a_1J + a_2 = 0 , \quad (4)$$

$$k_p = (1/k)(4a_0J^3 - 3a_1J^2 + 2a_2J + a_3), \quad (5)$$

$$k_i = (1/k)(-a_0J^4 + a_1J^3 - a_2J^2 - a_3J) + k_pJ. \quad (6)$$

The expression (4) is a function of object's parameters and unknown value of stability degree J. Solved the expression (4) it is determined the optimal value of stability degree J_{opt} , which presented the smallest positive and real root or the real positive part of complex root.

From expressions (5) and (6) were determined the optimal values of parameters k_p and k_i of PI controller.

In the case of tuning parameters of PID controller using the MSD method it was applying the algebraic expressions, which are the analytical expressions

$$4a_0J - 3a_1 = 0, \quad (7)$$

$$k_p = (1/k)(4a_0J^3 - 3a_1J^2 + 2a_2J + a_3) + 2k_dJ, \quad (8)$$

$$k_i = (1/k)(-a_0J^4 + a_1J^3 - a_2J^2 - a_3J) - k_dJ^2 + k_pJ, \quad (9)$$

$$k_d = (1/k)(-6a_0J^2 + 3a_1J - a_2). \quad (10)$$

The expressions (7) is a function of object's parameters and unknown value of stability degree J. Solved the expressions (7) it is determined the optimal value of stability degree J_{opt}, which presented the smallest positive and real root or the real positive part of complex root.

From expressions (8), (9) and (10) were determined the optimal values of parameters k_p, k_i and k_d of PID controller.

To verify the performance of control system with tuning parameters for the optimal values of the J_{opt} it was made the simulation. If the performance of control system are not satisfied it is proposed to tune controllers using the iterative procedure.

The tuning parameters of P, PI and PID controller - k_p , k_i and k_d are the function of known parametrs of control object and of the unknown value J stability degree of control system: $k_p=f(J)$, $k_i=f(J)$ and $k_d=f(J)$ (view relations (3), (5), (6), (8), (9) and (10)). Based on these relations in the case of known object's parameters and in the case of variation stability degree $J \geq 0$ in the strict limits it was made the respectivily calculations and obtained the dependences $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$ for P, PI and PID controllers.

To verify the performance of control system with controllers' parameters tuning for the suboptimal values of J it was made the simulation. If the performance don't sutisfy the imposed performance, it will be in the iterative process choose the other sets of values of controllers parameters from curves $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$ the procedure will repeat until the performance of system will be satisfied.

3. APPLICATION AND COMPUTER SIMULATION

To show the efficiency of the proposed algorithm for tuning the typical controllers P, PI, PID to the model object (1) which has the following parameters k =1, a_0=1, a_1=9, a_2=31, a_3=41 and it will be using the procedure exposes above for tuning typical controllers.

It will be presented two variants of calculating the controllers parameters.

Variant 1. Doing the respectively calculations in conformity with the elaborated algorithm for the given object are obtained the following results:

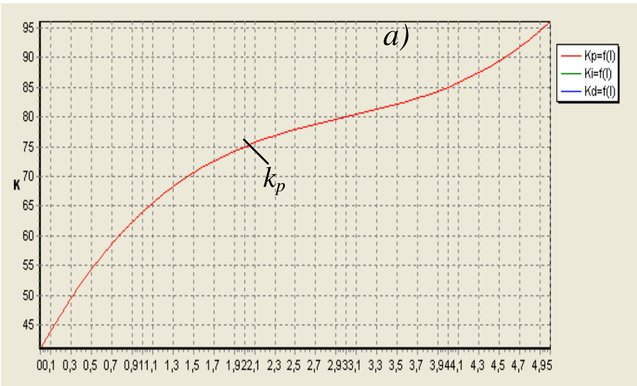
- for the P controller:
  J_{opt}=3, k_p=80;
- for the PI controller:
  J_{opt}=2.25, k_p=89.375, k_i=28.793 or T_i=0.0347 s ;
- for the PID controller:
  J_{opt}=2.25, k_p=86.5625, k_i=31.957 or T_i=0.0313, k_d=-0.625.

The component D from PID controller has the negative and small value, that's why the action of PID controller is equivalent with action of PI controller, that demonstrate that the values of P and I components of PID controller have a very similar values of components P and I of PI controller.

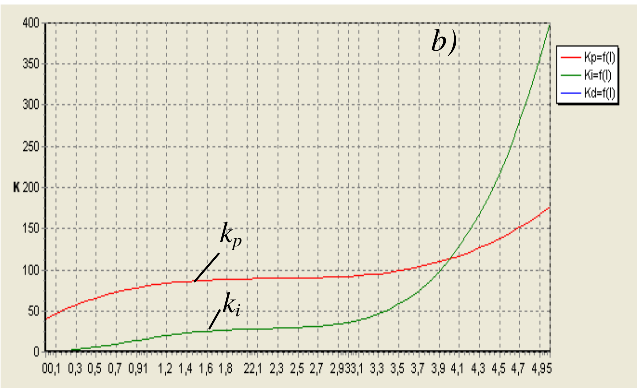
Variant 2. For the relations $k_p=f(J)$; $k_p=f(J)$ and $k_i=f(J)$; $k_p=f(J)$, $k_i=f(J)$ and $k_d=f(J)$ of P, PI and PID controllers (see relations (3), (5) and (6), (8), (9) and (10)) with the known parameters values of object and at the variation of the stability degree J in the respectively limits the iterative calculations were made and the obtained results are presented in the figure 2, *a, b, c*. Controller PID can not be tuning, because the curve $k_d=f(J)$ at the variation of the J has just the negative values (figure 2, c).

The sets of values J - k_p for the P controller (table 1);

J - k_p, k_i and for the PI controller (table 2) were chosen to analyze the set of performance of control system with P, PI and PID controllers from figure 2.



The value of stability degree



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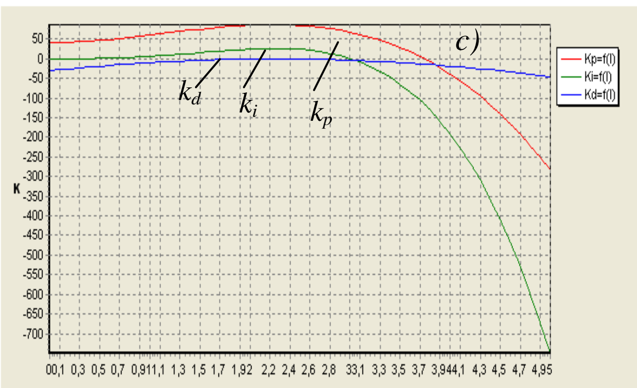

The value of stability degree

Fig. 2. The dependence of P, PI, PID controllers' parameters of the stability degree value.

For verify the obtained results in case of tuning controllers P, PI, PID to the model object (1) using the GMS it was made the computer simulation of the control system in MATLAB (figure 3).

Table 1. **The values of the P controller's parameters**

Nr.	J	k_p
1	0.32	60
2	0.7	70
3	1.3	75
4	**2**	**80**
5	3	90

Table 2. **The values of the PI controller's parameters**

Nr.	J	k_p	k_i
1	0.36	60	3.23
2	0.62	70	8.07
3	1	80	16
4	**2.25**	**89.375**	**28.793**
5	3.3	95	48

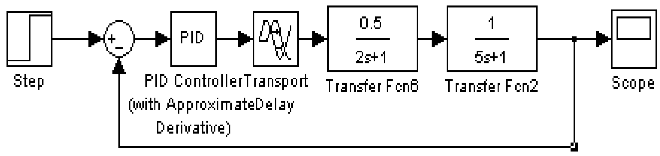

Fig. 3. Simulation diagrams of the control system.

The obtained results of computer simulation are presented in the figure 4, *a*, *b*: in the figure 4, *a* are presented the transition process of control system with P controller tuning after MSD method (curve 1-5); in the fig. 4, *b* are presented the transition process of control system with PI controller tuning after MSD method (curve 1-5), curve 6 - the transition process of control system with PI controller eguivalent PID controller tuning after MSD method and curve 7 the transition process of control system with PI controller optimization in MATLAB where k_p = 119.212 and k_i = 8.242.

The number of the curves it is coincided with the number of the values of the set's variant from tables 1 and 2 (the marked position 4 from tables 1, 2 present the optimal values).

Analyzing the obtained transition process of the control system with P, PI and PID controller tuning, the obtained performances of the CS are presented in the table 3.

It can be observed from figure 4 that for the optimal stability degree J_{opt} the sets of values of tuning parameters of P and PI controller are maximal for the object's parameters (1).

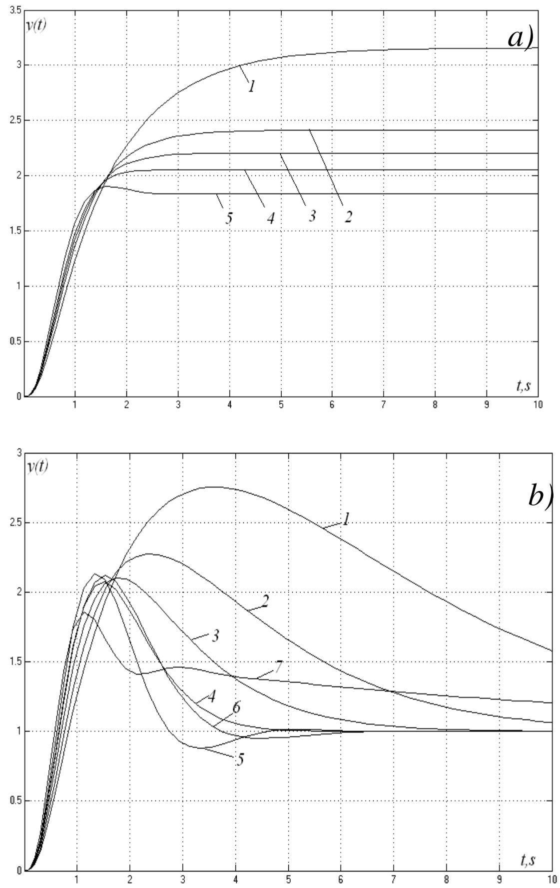

Fig. 4. The transient processes of control systems.

Analyzing the performance of the control system from figure 4, *a*, *b* it was observed that **the optimal performances** were obtained for curves 5 (PI controller eguivalent PID controller) with J_{opt}=2.25, ε = +5 % and t_r=3,44 s, which correspond with stability degree calculated from relation (4), but for the other transitional processes the tuning time presented to table 3.

The control system with PI controller optimization in MATLAB has the higher performance then performance of control system with PI controller tuning by MSD method.

Table 3. **Performance of control system**

Perform. of CS		ε, %	t_c, s	σ,%	t_r, s	λ
GMS Method	P	5	1.5		1.5	
	PI	5	0.62	110	4.04	1
	PID	5	0.63	114	3.44	1
MATL.	PI	5	0.55	87	22.9	1

For the control system with PI controller (parameters k_p = 89.375 and k_i = 28.793) tuning by MSD method, which the transition process (curve 7) are presented in the figure 4, were analyzed the distribution of poles of characteristic equation of the control system in the complex plan which were calculated in the MATLAB and presented in the figure 5.

In the figure 5 are presented the domination poles: 1 – for the control system with PI controller tuning by MSD method, the minimal pole has value $p_{2,3}$ = -2.25 ± 0.791, $p_{4,5}$ = -2.25±0.007071 for the control system with PI controller eguivalent PID controller tuning by MSD method, the value of zero for the both algorithms is $z=-0.322$.

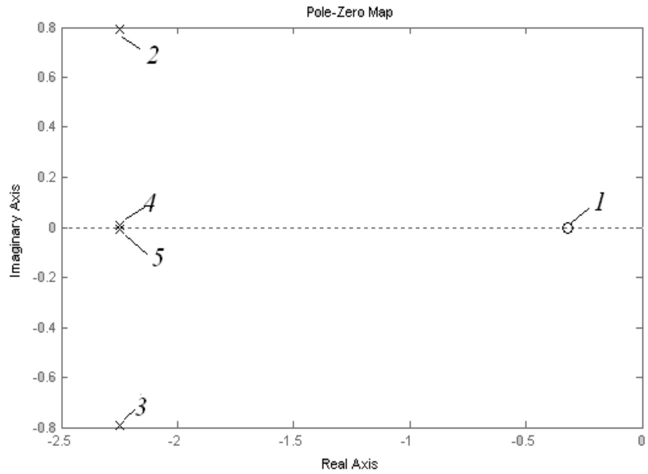

Fig. 5. The distribution of the characteristic equation's poles.

Analyzing the distribution of poles of characteristic equations of control system with PID tuning by MSD method it can be observed that the relative stability of the control system with P, PI and PID controllers tuning by MSD method has the reserve of stability much higher than the reserve of stability of the control system with PI controller optimization in MATLAB.

It follows that the robustness of control system with PID controller tuning by MSD method in the case of the variation of parameters of the control object (1) is higher than the robustness of control system with PI controller optimizion in MATLAB.

4. CONCLUSION

As a result of the study, the following conclusion can be made:

- It is proposed the iterative (grafo-analitical) method of tuning P, PI, PID controllers to the model objects with inertia and nonminimal phase which permitted to obtain the settled performance.
- For the control system with PI controller tuning by MSD method, the transition process of system is osscilating and optimal for the given values of object (view the curve 4, fig. 4, *a*, *b*). For the control system with PID controller tuning by MSD method the transition process of system are osscillating for the

given values of the object (view the curve 5 fig. 3, *b, c*), but with performance much better then the performance the transition process of control system with controller PI optimization in MATLAB.

- Analyzing the transition processes of the control system with *PID* controller from fig. 7 it can be seen that making the variation of the degree stability of the control system were obtained the sated performance of the control system (curve 3, fig. 8). In this case it is necessary to make the iterative calculations for the difference value sets of the J- k_p , k_i, k_d for the PID controller.

- Control system with PID controller tuning by MSD method has t_r the reserve of stability much higher than the reserve of stability of the control system with PI controller optimization in MATLAB.

- The robustness of control system with PI, PID controller tuning by MSD method in the case of the variation of parameters of the control object (1) is higher than the robustness of control system with PI controller optimization in MATLAB.

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