BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 66 (70), Numărul 1, 2020
Secţia
ELECTROTEHNICĂ. ENERGETICĂ. ELECTRONICĂ

SIZING OF THE COGENERATION PLANTS BY THE OPTIMIZED RECTANGLE METHOD

BY

VALENTIN ARION, CĂLIN NEGURA and CONSTANTIN BOROSAN[*]

Technical University of Moldova, Faculty of Power and Electrical Engineering,
Department of Energy, Chişinău, Republic of Moldova

Received: October 13, 2020
Accepted for publication: December 8, 2020

Abstract. This paper addresses the issue of sizing cogeneration plants. A simple analytical expression has been obtained for cogeneration plants sizing and determining their share to cover the thermal load of either a consumption node or a heat supply system. The resulted expression is based on the maximum rectangle method and allows rapid and accurate identification of the optimal share of cogeneration. Unlike the traditional application of the maximum rectangle method, which requires analyzing several rectangles inscribed under the load curve and comparing their surface, while the new proposed formula allows finding the optimal share of cogeneration by a single direct calculation. This approach is based on the analytical description of the load curve of the consumption node / area, using a power function, known as the Sochinsky-Rossander function. It should be noted that the resulted solution based on this method does not guarantee matching the optimal solution from an economic point of view.

Keywords: load duration curve; load factor; maximum rectangle method, non-uniformity factor; Sochinsky-Rossander power function.

[*]Corresponding author: *e-mail*: constantin.borosan@tme.utm.md

1. Introduction

A cogeneration plant represents a complex and expensive infrastructure. Its specific investment cost is much higher compared to the hot water boilers. Despite this, cogeneration is widely promoted due to the possibility of significant reduction of primary energy resources consumption (fossil fuels) in a combined heat and power production (co-generation) mode and, respectively, greenhouse gas emissions reduction (Directive 2012/27/EU; Ministerial Council Decision D/2015/08/MC-EnC; EU Staff Working Document, 449 final, 2013; Law no.139, 2018; Law no.92, 2014; Law no.128/2014). The technology for cogeneration of electricity (*W*) and heat (*Q*) is not a new one - it is over a century old, but so far there is no widely recognized methodology for sizing the installed capacity.

Among the existing recommendations in the literature regarding the thermal capacity choice of the urban cogeneration plants (q_{nom}^{CHP}) there is often a proposal to accept the q_{nom}^{CHP} value at the level of the minimum annual thermal load $q_{\min}^{load}$ of the considered consumption node (conservative option - $q_{nom}^{CHP} = q_{min}^{load}$) (Office of Environment and Heritage - NSW, 2014).

It must be recognized that this solution is safe and attractive; it involves less risk and guarantees two important things:

– the production of the *W* and *Q* energy will be at high energy efficiency (Directive 2012/27/EU; Ministerial Council Decision D/2015/08/MC-EnC; EU Staff Working Document, 449 final, 2013; Office of Environment and Heritage - NSW, 2014),

– the use of cogeneration will ensure a high level of economic profitability.

If there is a wish to increase the level of profitability of the cogeneration plant, one should consider other solutions, characterized by higher capacities, $q_{nom}^{CHP} > q_{min}^{load}$. Thus, over time, a comparatively simple way for sizing the cogeneration plants has been reached, now known as the maximum rectangle (MR) method, which has soon expanded (Haeseldonckx *et al.*, 2007; Shaneb *et al.*, 2011; Volkova and Siirde, 2011; Zapata Riveros, 2015; Magnani *et al.*, 2016; Yu *et al.*, 2017; Li *et al.*, 2017; Sanaye and Khapaay, 2014; Ebrahimi and Keshavarz, 2012; Ruan *et al.*, 2016; Gu *et al*., 2012).

In this paper, an essential improvement is proposed for finding the optimal solution for sizing the cogeneration plants based on the application of the maximum rectangle criterion.

2. The Essence of the Maximum Area Rectangle Method

The maximum rectangle method was proposed (Haeseldonckx *et al.*, 2007) for the purpose of the cogeneration plants sizing. Its essence consists in inscribing under the annual heat load duration curve (H-LDC) of the considered consumption node a rectangle with the largest possible area (Fig. 1); the height of this rectangle will indicate the thermal capacity of the cogeneration unit.

In order to identify the rectangle with the largest area, a search process is usually organized, in several steps (Yu *et al.*, 2017); at each step i it is considered a rectangle with a height x_i, $i = 1,I$ and a width τ_{x_i}, respectively, determined by the point of intersection of the horizontal x_i with the load curve (Fig. 1). Thus, at step 1 one will accept a value $x_1 = q_{min*}$ (height of the rectangle) for which, at the intersection of the horizontal line x_1 with H-LDC, on the time axis, results the duration τ_{x_1} (width of the rectangle; point 1, Fig. 1), and finally, the area S_1 of the respective rectangle is determined as $S_1 = x_1 \cdot \tau_{x_1}$. At step 2 the value x_2 is accepted, and on the load duration curve the point 2 and the duration τ_{x_2} are identified, respectively, allowing later on to calculate the area S_2, $S_2 = x_2 \cdot \tau_{x_2}$. The same procedure is repeated for all other accepted values of x_i (Fig. 1). Following the comparison of the considered solutions, one will find such a value of $x_{opt,MR}$ yielding a maximum area of the rectangle ($x_{opt,MR} \to S_{max}$); the more alternative solutions are considered, the more accurate the final solution will yield.

Thus, the optimal solution for the thermal capacity of the cogeneration plant ($q^{CHP}_{nom,opt}$), resulting from the application of the MR method is: $q^{CHP}_{nom,opt} = x_{opt,MR} \cdot q^{load}_{max*}$. Here, the value of $x_{opt,MR}$ obviously expresses the *share of cogeneration* in the maximum annual heat load of the consumer, and the value $(1 - x_{opt,MR})$ - the *share of heat only boilers*, respectively.

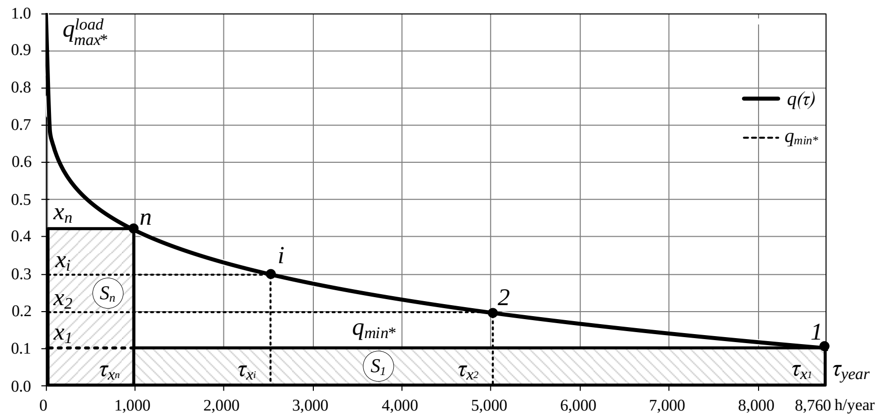

Fig. 1 − Illustration of the MR method applied for the determination of the optimal share of the cogeneration plant.

It is easy to notice that for analytical description of the load curve, based on the considered criterion ($x_{opt,MR} : S \rightarrow max$), the finding of the optimal solution can be simplified, as it would avoid the need to generate and evaluate alternative variants. This aspect will be addressed beneath.

3. The Analytical Model Applied to the Load Duration Curves

The Sochinsky-Rossander power function is used to model the energy consumption widely, which has the form (Arion and Negura, 2018) for the heat load duration curve:

$$q(\tau) = q_{max} \cdot [1 - (1 - q_{min} / q_{max}) \cdot (\tau / \tau_{year})^{\beta}] \quad (1)$$

and in the case of a normalized curve it is:

$$q_*(\tau_*) = 1 - (1 - q_{min*}) \cdot \tau_*^{\beta} \quad (2)$$

where: q_{max} and q_{min} represents the maximum and minimum value of the load during that year. Asterisk symbol $*$ stands for the relative value of the parameter.
q_{min*} – the minimum relative value of the load, $q_{min*} = q_{min} / q_{max}$;
τ – the current value of the time; $\tau_{year} = 8\,760\ h / year$;
β – the power factor, the non-uniformity factor of the load duration curve:

$$\beta = (q_{med*} - q_{min*}) / (1 - q_{med*}) \text{ or } \beta = (T_M - \tau_{year} \cdot q_{min*}) / (\tau_{year} - T_M) \quad (3)$$

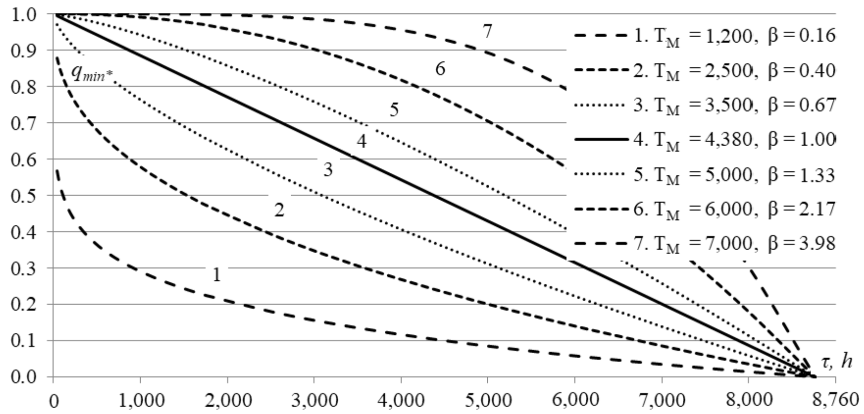

Fig. 2 − Examples of load duration curve shape (Sochinsky-Rossander model) for the case $q_{min*} = 0$.

4. The Optimized Rectangle Method

The load curve modelling using an approximation function allows to express the area of the rectangle, considered in the above method, by the parameters of the load curve and the height x of the rectangle. In addition, having an analytical description of the area S, the rectangle with the maximum area and its height, $x_{opt,MR}$, can be identified in a single step.

The rectangle area, inscribed in the space below the load duration curve:

$$S = q_{nom}^{CHP} \cdot \tau_x \tag{4}$$

where the calculation expression for the duration τ_x, resulting from Eq. (2) of the H-LDC, is:

$$\tau_x = \tau_0 \cdot (1 - x)^{1/\beta} \tag{5}$$

where

$$\tau_0 = (1 - q_{min})^{-1/\beta} \tag{6}$$

Taking into account the fact that the thermal capacity q_{nom}^{CHP} can be expressed as a share x^{CHP} of the maximum annual heat load of the consumption node - $q_{nom}^{CHP} = q_{max}^{load} \cdot x^{CHP}$, for area S, there is:

$$S = q_{max}^{load} \cdot x^{CHP} \cdot \tau_x \tag{7}$$

If in (7) the duration τ_x is substituted by the formula (5), the expression sought for the area of the rectangle is obtained:

$$S = q_{max}^{load} \cdot x^{CHP} \cdot \tau_0 \cdot (1 - x^{CHP})^{1/\beta} \quad (8)$$

The optimal value of the height of the maximum rectangle $x_{opt,MR}^{CHP}$, which corresponds to the criterion $S = q_{nom}^{CHP} \cdot \tau_x \rightarrow max.$, is determined by cancelling the first-order derivative of the expression (8) as a function of x^{CHP}:

$$\partial S / \partial x^{CHP} = 0 \text{ for } \partial^2 S / \partial^2 x^{CHP} \geq 0 \quad (9)$$

From condition (9), finally, results the calculation expression of the optimal share of cogeneration $q_{opt,MR}^{CHP}$:

$$q_{opt,MR}^{CHP} = \beta / (1 + \beta) \quad (10)$$

Thus, it is demonstrated that the optimal solution, resulting from the application of the criterion „rectangle area → max.”, can be determined analytically in a single step, by a direct calculation (formula (10)) - without generating and evaluating alternative variants.

In addition, the expression (10) turns out to be extremely simple and demonstrates that $q_{opt,MR}^{CHP}$ depends only on the non-uniformity factor β of the load curve, so only on the load duration curve shape (which was obvious from the start!).

Another important observation can be pointed out from the following expression - if the minimum annual load of the consumption node is equal to zero ($q_{min}^{load} = 0$), from (10), taking into account (3), it results:

$$q_{opt,MR}^{CHP} = \beta / (1 + \beta) = T_{M*} = LF_Q \quad (11)$$

where T_{M*} represents the relative value of the maximum heat load usage time ($T_{M*} = T_M / \tau_{year}$), also known as the *load factor* (LF) of the consumption node.

Beneath is a small numerical example, which illustrates the abovesaid.

5. Numerical Example

Table 1 presents the values of β - factor of the H-LDC non-uniformity, and in Table 2 - the values of the optimal share of cogeneration $q_{opt,MR}^{CHP}$ for multiple combinations of parameters q_{min*} and T_{M*} of the load curve.

Table 1
The values of β - factor of the H-LDC non-uniformity according to the parameters q_{min*} and T_M

q_{min*}, r.u.	Maximum heat load usage time T_M									
	h/year	**876**	**1,752**	**2,628**	**3,504**	**4,380**	**5,256**	**6,132**	**7,008**	**7,884**
	r.u.	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0		*0.11*	*0.25*	*0.43*	*0.67*	*1.00*	*1.50*	*2.33*	*4.00*	*9.00*
0.1		*0*	*0.13*	*0.29*	*0.50*	*0.80*	*1.25*	*2.00*	*3.50*	*8.00*
0.2			*0*	*0.14*	*0.33*	*0.60*	*1.00*	*1.67*	*3.00*	*7.00*
0.3				*0*	*0.17*	*0.40*	*0.75*	*1.33*	*2.50*	*6.00*
0.4					*0*	*0.20*	*0.50*	*1.00*	*2.00*	*5.00*
0.5						*0*	*0.25*	*0.67*	*1.50*	*4.00*
0.6							*0*	*0.33*	*1.00*	*3.00*
0.7			β					*0*	*0.50*	*2.00*
0.8									*0*	*1.00*
0.9										*0*

Table 2
The values of the optimal share of cogeneration $q_{opt,MR}^{CHP}$

q_{min*}, r.u.	Maximum heat load usage time T_M									
	h/year	**876**	**1,752**	**2,628**	**3,504**	**4,380**	**5,256**	**6,132**	**7,008**	**7,884**
	r.u.	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0		*0.100*	*0.200*	*0.300*	*0.400*	*0.500*	*0.600*	*0.700*	*0.800*	*0.900*
0.1		*0*	*0.111*	*0.222*	*0.333*	*0.444*	*0.556*	*0.667*	*0.778*	*0.889*
0.2			*0*	*0.125*	*0.250*	*0.375*	*0.500*	*0.625*	*0.750*	*0.875*
0.3				*0*	*0.143*	*0.286*	*0.429*	*0.571*	*0.714*	*0.857*
0.4					*0*	*0.167*	*0.333*	*0.500*	*0.667*	*0.833*
0.5						*0*	*0.200*	*0.400*	*0.600*	*0.800*
0.6							*0*	*0.250*	*0.500*	*0.750*
0.7		$q_{opt,MR}^{CHP} = \beta / (1+\beta)$						*0*	*0.333*	*0.667*
0.8									*0*	*0.500*
0.9										*0*

These data can be presented in more details (Tables 3 and 4) for the value ranges of the parameters q_{min*} and T_M characteristic of the heat load curves in the residential and tertiary sectors.

Table 3

Residential and tertiary consumption: β - factor of the H-LDC non-uniformity values for realistic combinations of q_{min*} and T_M

q_{min*}, r.u.	Maximum heat load usage time T_M										
	h/year	500	1,000	1,500	2,000	2,500	3,000	3,500	4,000	4,500	5,000
	r.u.	0.06	0.11	0.17	0.23	0.29	0.34	0.40	0.46	0.61	0.57
0		0.06	0.13	0.21	0.30	0.40	0.52	0.67	0.84	1.06	1.33
0.05		0.01	0.07	0.15	0.23	0.33	0.44	0.58	0.75	0.95	1.21
0.08		0	0.04	0.11	0.19	0.29	0.40	0.53	0.69	0.89	1.14
0.10			0.02	0.09	0.17	0.26	0.37	0.50	0.66	0.85	1.10
0.12				0	0.06	0.14	0.23	0.34	0.47	0.62	0.81
0.15					0.03	0.10	0.19	0.29	0.42	0.56	0.75
0.18					0	0.06	0.15	0.25	0.37	0.51	0.69
0.20			β			0.04	0.12	0.22	0.33	0.47	0.65
0.22						0.01	0.09	0.19	0.30	0.44	0.60
0.25						0	0.05	0.14	0.25	0.38	0.54

From Table 3 one might notice that for the condition $q_{min*} = 0$, $q_{opt,MR}^{CHP} = LF_Q = T_{M*}$.

Table 4

Residential and tertiary consumption: $q_{opt,MR}^{CHP}$ values for realistic combinations of q_{min*} and T_{M*}

q_{min*}, r.u.	Maximum heat load usage time T_M										
	h/year	500	1,000	1,500	2,000	2,500	3,000	3,500	4,000	4,500	5,000
	r.u.	0.06	0.11	0.17	0.23	0.29	0.34	0.40	0.46	0.61	0.57
0		0.06	0.11	0.17	0.23	0.29	0.34	0.40	0.46	0.51	0.57
0.05		0.01	0.07	0.13	0.19	0.25	0.31	0.37	0.43	0.49	0.55
0.08		0	0.04	0.10	0.16	0.22	0.29	0.35	0.41	0.47	0.53
0.10			0.02	0.08	0.14	0.21	0.27	0.33	0.40	0.46	0.52
0.12			0	0.06	0.12	0.19	0.25	0.32	0.38	0.45	0.51
0.15				0.02	0.09	0.16	0.23	0.29	0.36	0.43	0.50
0.18				0	0.06	0.13	0.20	0.27	0.34	0.41	0.48
0.20		$q_{opt,MR}^{CHP} = \beta / (1+\beta)$			0.04	0.11	0.18	0.25	0.32	0.39	0.46
0.22					0.01	0.08	0.16	0.23	0.30	0.38	0.45
0.25					0	0.05	0.12	0.20	0.28	0.35	0.43

For a considered consumption node, the dependence of the value of the cogeneration share on the value of the factor of the H-LDC non-uniformity β - is presented in Table 5 and Fig. 3.

Table 5

The values of the optimal share of cogeneration $q_{opt,MR}^{CHP}$ *depending on the value of* β

β	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
$q_{opt,MR}^{CHP}$	*0.09*	*0.17*	*0.23*	*0.29*	*0.33*	*0.38*	*0.41*	*0.44*	*0.47*	*0.50*

Continuation

β	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**1.6**	**1.7**	**1.8**	**1.9**	**2**
$q_{opt,MR}^{CHP}$	*0.52*	*0.55*	*0.57*	*0.58*	*0.60*	*0.62*	*0.63*	*0.64*	*0.66*	*0.67*

For the examples of load curves, presented in Fig. 2, Table 6 provides the values of the sides of the maximum area rectangles inscribed under these curves - the values $q_{opt,MR}^{CHP}$ and τ_x, and Fig. 4 illustrates the graphical aspect of the optimal solution corresponding to the considered load curves.

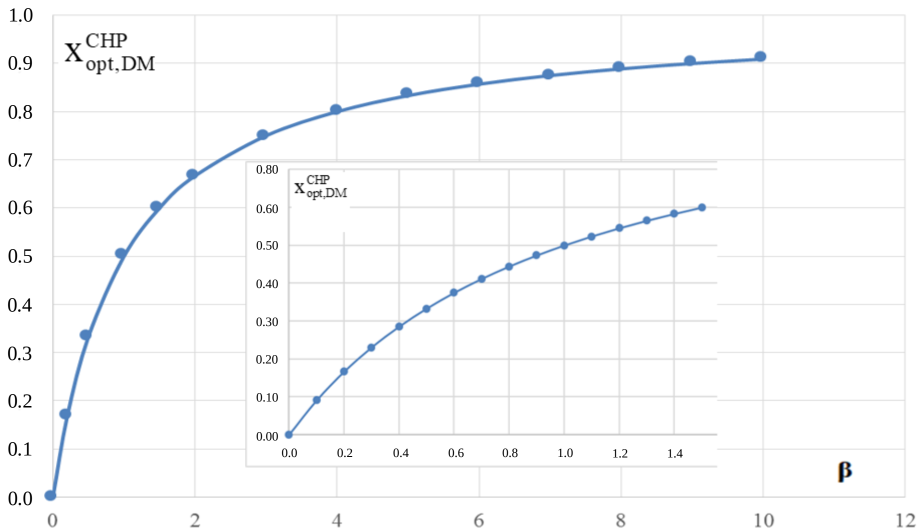

Fig. 3 – Dependence of the x_{opt} on the value of the β - factor of the H-LDC non-uniformity.

Table 6

The optimal share of cogeneration $q_{opt,MR}^{CHP}$ *depending on the values of* β *- factor of the H-LDC non-uniformity*

No. H-LDC	T_M	$T_{M*} = T_M / \tau_{year}$	β	$q_{opt,MR}^{CHP}$	τ_x
1	*1,200*	*0.14*	*0.16*	*0.14*	*3,463*
2	*2,500*	*0.29*	*0.40*	*0.29*	*3,777*
3	*3,500*	*0.40*	*0.67*	*0.40*	*4,070*
4	*4,380*	*0.50*	*1.00*	*0.50*	*4,380*
5	*5,000*	*0.57*	*1.33*	*0.57*	*4,637*
6	*6,000*	*0.68*	*2.17*	*0.68*	*5,150*
7	*7,000*	*0.80*	*3.98*	*0.80*	*5,851*

6. Discussions and Conclusions

The criterion of the maximum area rectangle, applied for cogeneration plants sizing, certainly contains a rational grain: from the point of view of the economic interest, one desires a higher capacity of the cogeneration than its minimum annual plant load, but not so high as to „undermine" the economic and financial feasibility of the investment!

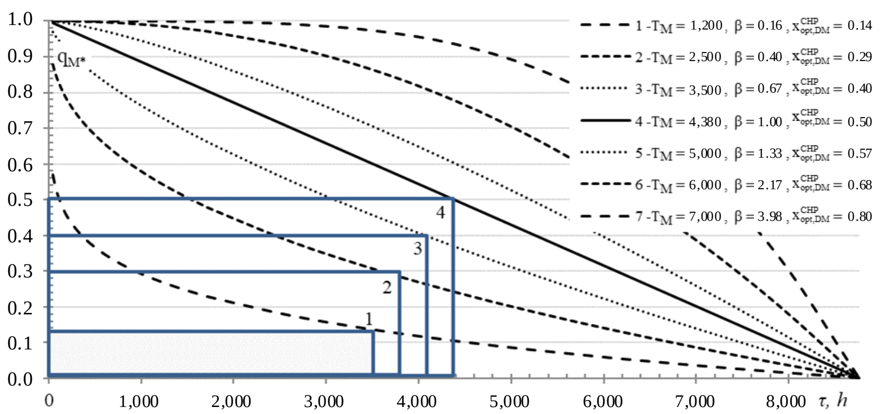

Fig. 4 − Examples regarding the allure of the annual load curve for the case $q_{min*} = 0$, as well as rectangles of maximum area inscribed below the considered load curves.

Herewith, obviously remains the question whether the solution $q_{opt,MR}$, found thereby, is close to the optimal solution $q_{opt,ec}$, which results on the basis of the economic criterion. This question is relevant in the context of cogeneration installations sizing based on the MR criterion, but it falls outside the scope of this article and is to be addressed in a separate paper.

REFERENCES

Arion V., Negura C., *Improving the Model of Head Load Duration Curve*, 14th Regional Energy Forum, Vox Maris Grand Resort, Costineşti, Romania, 2018.

Ebrahimi M., Keshavarz A., *Climate Impact on the Prime Mover Size and Design of a CCHP System for the Residential Building*, Energy and Buildings, **54**, 2012, 283-289.

Gu Q., Ren H., Gao W., Ren J., *Integrated Assessment of Combined Cooling Heating and Power Systems Under Different Design and Management Options for Residential Buildings in Shanghai*, Energy and Buildings, **51**, 2012, 143-152.

Haeseldonckx D., Peeters L., Helsen L., D'haeseleer W., *The Impact of Thermal Storage on the Operational Behaviour of Residential CHP Facilities and the Overall CO_2 Emissions*, Renewable and Sustainable Energy Reviews, **11**, *6*, 2007, 1227-1243.

Li B., Robin Roche R., Damien Paire D., Miraoui A., *Sizing of a Stand-Alone Microgrid Considering Electric Power, Cooling/Heating, Hydrogen Loads and Hydrogen Storage Degradation*, Applied Energy, **205**, 2017, 1244-1259.

Magnani S., Pezzola L., Danti P., *Design Optimization of a Heat Thermal Storage Coupled with a Micro-CHP for a Residential Case Study*, Energy Procedia **101**, 2016, 830-837.

Ruan Y., Liu Q., Li Z., Wu J., *Optimization and Analysis of Building Combined Cooling, Heating and Power (BCHP) Plants with Chilled Ice Thermal Storage System*, Applied Energy, **179**, 2016, 738-754.

Sanaye S., Khakpaay N., *Simultaneous Use of MRM (Maximum Rectangle Method) and Optimization Methods in Determining Nominal Capacity of Gas Engines in CCHP (Combined Cooling, Heating and Power) Systems*, Energy, **72**, 2014, 145-158.

Shaneb O.A., Coates G., Taylor P.C., *Sizing of Residential μCHP Systems*, Energy and Buildings, **43**, *8*, 2011, 1991-2001.

Volkova A., Siirde A., *The Use of Thermal Energy Storage for Energy System Based on Cogeneration Plant*, In: Recent Researches in Geography, Geology, Energy, Environment and Biomedicine: The 5th International Conference on Energy and Development-Environment-Biomedicine, Greece, Corfu, 14-16 July, 2011. Corfu: WSEAS, 2011, 71-75, ISBN 9781618040220.

Yu D., Meng Y., Yan G., Mu G., Li D., Le Blond S., *Sizing Combined Heat and Power Units and Domestic Building Energy Cost Optimisation*, 2017, Energies, **10**, *6*, 771.

Zapata Riveros J. V, *Facilitating the Integration of Renewable Energy Through Combined-Heat-and-Power Flexibility*, 2015, Web, 187 p. PhD Thesis in Engineering Science, Leuven.

*** *Decision of the Ministerial Council of the Energy Community D/2015/08/MC-EnC: on the Implementation of the Directive 2012/27/EU of the European Parliament and of the Council of 25 October 2012 on Energy Efficiency*.

*** *Directive 2012/27/EU of the European Parliament and of the Council of 25 October 2012 on Energy Efficiency*, https://www.legis.md/.

*** Energy Saver, *Cogeneration Feasibility Guide*, 138 p. Office of Environment and Heritage NSW, 2014, ISBN 9781743592380.

*** *Guidance Note on Directive 2012/27/EU on Energy Efficiency*, Article 14: Promotion of Efficiency in Heating and Cooling, SWD (2013) 449 final, Brussels, 2013, 24.

*** Legea Republicii Moldova nr.92/2014 *cu privire la energia termică și promovarea cogenerării*, https://www.legis.md/.

*** Legea Republicii Moldova nr.128/2014 *privind performanța energetică a clădirilor*, https://www.legis.md/.

*** Legea Republicii Moldova nr.139/2018 *cu privire la eficiența energetică*, https://www.legis.md/.

DIMENSIONAREA INSTALAȚIILOR DE COGENERARE A ENERGIEI PRIN METODA DREPTUNGHIULUI OPTIMIZAT

(Rezumat)

În această lucrare este abordată problematica dimensionării centralelor de cogenerare. A fost obținută o expresie analitică simplă de dimensionare a cotei instalațiilor de cogenerare pentru acoperirea sarcinii termice fie a unui nod de consum, fie a unui sistem de alimentare cu energie termică. Expresia obținută se bazează pe metoda dreptunghiului de arie maximă și permite identificarea rapidă și precisă a cotei optime a cogenerării. Spre deosebire de modul tradițional de aplicare a metodei dreptunghiului, care presupune analiza mai multor dreptunghiuri înscrise sub curba de sarcină și compararea suprafeței acestora, formula obținută permite găsirea cotei optime a cogenerării printr-un singur calcul direct. La baza acestei abordări este pusă descrierea analitică a curbei de sarcină a nodului / zonei de consum, cu aplicarea unei funcții putere, cunoscută sub denumirea funcția Sochinsky-Rossander. De menționat, că soluția ce rezultă în baza aplicării acestei metodei nu garantează coincidență cu soluția optimă din punct de vedere economic.