

Energy Conversion and Management 44 (2003) 1039-1051



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## Integral measures of electric power distribution networks: load–length curves and line-network multipliers

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Received 18 January 2002; accepted 23 April 2002

## Abstract

This paper describes an integral (macroscopic) approach to the evaluation and maximization of performance in the design of networks for the distribution of electric power. The method recognizes the hierarchical structure of tree shaped networks and accounts for all the consumption nodes, links (lines) and loads carried by each line. The lines that carry the same load form one group. The total length of the lines belonging to one group is calculated. The total length of all the lines is the sum of the total lengths calculated for each group. This analysis makes it possible to represent one network as a curve in the load– length plane. Next, the moment of the load at one consumption node is defined as the load times the direct distance from the source to the node. The global load moment is the sum of all the individual load moments. The global load moment is proportional to the area trapped above the load–length curve of the network. It is shown that this area must be minimized in order to minimize the discounted total cost of the network. Six competing distribution networks are compared on this integral basis. Networks that are tree shaped and sufficiently complex perform at nearly the same (high) level. The robustness exhibited by tree shaped power distribution networks is similar to the robustness of other tree flow structures (fluid, heat, goods, people) in animate, inanimate and engineered flow systems in accordance with constructal theory. © 2002 Published by Elsevier Science Ltd.

Keywords: Constructal design; Power distribution; Tree networks; Topology optimization; Economic optimization

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<sup>0196-8904/02/\$ -</sup> see front matter @ 2002 Published by Elsevier Science Ltd. PII: S0196-8904(02)00108-5

## Nomenclature

- $C_{\rm m}$  specific cost of load moment (\$/A km)
- $C'_{\rm pw}$  total cost of demand and energy losses (\$/kW)
- $E_{\Sigma}$  factor, Eq. (10)
- $I_i$  current load of lines in group i (A)
- $I_0$  elemental current load (A)
- $j_{ec}$  economic current density (A/mm<sup>2</sup>)
- $k_{\rm I}$  line specific investment cost (\$/mm<sup>2</sup> km)
- $K_{\rm m}$  line-network multiplier, Eq. (8)
- $l_0$  elemental length (km)
- $L_i$  total length of lines in group *i* (km)
- $L_{ij}$  length of one line in group *i* (km)
- $L_{i-s}$  direct distance between node *i* and source (km)
- M total network load moment (A km), Eq. (4)
- $M_v$  moment of load  $I_v$  at one node (A km)
- $M_0$  elemental load moment (A km)

*n* number of line groups

- $n_{\rm c}$  number of consumers on one side of a square territory, Fig. 14
- $n_i$  number of lines in group i
- *N* total number of nodes
- TCD discounted total cost (\$), Eq. (9)
- *S* area above load–length curve (A km)
- $S_i$  area associated with load moment  $M_i$  and lines loaded in range  $I_i \pm \Delta I_i/2$

Greek symbols

 $\alpha_{\rm B}$  book value parameter, Eq. (10)

- $\alpha_{\rm M}$  maintenance cost parameter, Eq. (10)
- $\rho$  resistivity ( $\Omega$  mm<sup>2</sup>/km)
- $\sum$  sum
- Subscripts i group of lines loaded with  $I_i$
- 0 elemental
- 0 elementai

## 1. Hierarchical tree shaped distribution networks

The performance of a distribution network that serves a large number of consumers can be described in many ways that refer to the quality of the delivered goods, cost, accessibility of delivery points, timeliness, reliability and network complexity. In the design optimization phase of a project, the large number and diversity of such criteria make it difficult to compare, on the same basis, several competing distribution structures. In this paper, we focus on a class of highly complex flow structures: the networks for distribution of electric power. We show that the per-

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