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STUDY OF THE ANALYTICAL DESCRIPTION OF NESTANDARD TOOTH PROFILE BY USING THE SYSTEM OF EQUATIONS

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Abstract: *The engineering complex study of the triad “gear-technology-transmission” has permitted to elaborate a new type of mechanical transmission – planetary precessional transmissions with multicouple gear. In this paper, the authors present the mathematic model of the multicouple gear. A computer program for doing this it is also elaborated. It is shown the calculus modalities and some teeth profile diagrams.*

Key words: *precessional transmission, multicouple gear, tooth profile.*

1. INTRODUCTION

The diversity of beneficiaries' requirements concern mechanical transmissions is reduced, specially, to the increase of reliability, efficiency and to the decrease of mass and dimensions. It becomes more and more difficult to satisfy the mentions requirements by updating partially the traditional transmissions. This problem can be solved by using new types of mechanical transmissions - planetary precessional transmissions [1,2]. The absolute multiplicity of the precessional gear (up to 100% simultaneously engaged teeth pairs) provides high bearing capacity and increased kinematic accuracy, reduced dimensions and mass. In addition, large kinematic possibilities (-8...-3600), reduced acoustic emission and solution of all technological issues open advantages that are very important for the utilization of planetary precessional transmissions in various areas of machine building. The authors elaborated a large number of diagrams concern planetary precessional transmissions (for reducers, multipliers and differentials), multiple gears for power and kinematic transmissions, gear processing methods and their control, the majority of works being patented with about **170 patents**. Know-how in the elaboration of the multicouple precessional gear, manufacturing technology and control methods, and a range of

precessional transmission diagrams belong to research team from the Technical University of Moldova.

The specific character of sphere-spatial (precessional motions of the precessional transmissions pinion makes impossible the utilisation of teeth classical involute profiles. This fact requires the elaboration of new profiles adequate to the sphere-spatial motion of pinion which would ensure high performances to the precessional transmission. Carrying out on the principle of the transfer function continuity and gear based on the principles of the transfer function continuity and gear multiplicity which aims to:

- the elaboration of the gear mathematics model with account of the peculiarities;
- the analytical description of teeth profiles by a system of parametric equations on spherical surface and normal teeth section for inner and plane gear;
- the determination by CAD of geometrical and cinematic parameters influence of the gear upon the teeth profiles shape and the justification of their rational limits of variation;
- the elaboration of the theoretical basis evaluation of teeth gear multiplicity in precessional transmissions;
- the definition of area of gear multiplicity existence by 100% teeth couples.

2. ELABORATION OF DIAGRAM OF PRECESSIONAL TRANSMISSIONS

The elaboration of working machines driving mechanisms is based on the diagram of precessional transmissions, presented in figure 1 [1]. The rotating motion of the crank

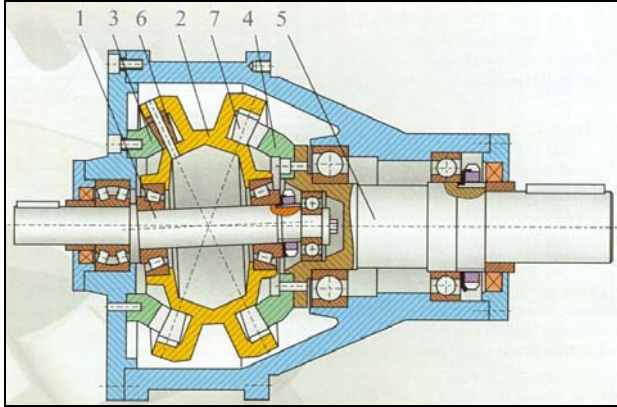


Fig. 1. Diagram of the planetary precessional transmission.

shaft 1 is transformed into sphere-spatial motion of the block pinion 2 with two teathed crowns 3 and 4, which are rolling without sliding on the immovable and driven toothed wheel teeth 5 and 6. Due to the minimum difference between the number of teeth $Z_5 = Z_3 - 1, Z_6 = Z_4 - 1, Z_3 = Z_4 + 1, 2, 3, \dots$ the transmission ratio is

$$i = \pm \frac{Z_3 Z_6}{Z_5 Z_4 - Z_3 Z_6}. \tag{1}$$

The teeth of crowns 3 and 4 are manufactured in the shape of conical rollers installed on axis having the possibility to rotate round them, and the teeth of central wheel 5 and 6 have non-standard convex-concave profile.

3. PARAMETRICAL EQUATION OF THE TOOTH PROFILE

In precessional gear the planet gear performs spherical-spatial motion around one fixed point [3,4]. It is known [Euler], that the body that performs spherical motion has three degrees of freedom. In theoretical mechanics, usually, the position of the body that performs precessional motion is defined by Euler angles.

The coordinates of the modified point D^m are:

$$\begin{aligned} X_D^m &= -\sin \delta \sin [Y_C^m \sin \theta + Z_C^m (1 - \cos \theta) \cos \psi]; \\ Y_D^m &= -Y_C^m \cos \delta + Z_C^m \sin \delta [\cos^2 \psi + \cos \theta \sin^2 \psi]; \\ Z_D^m &= -Y_C^m \sin \delta (\cos^2 \psi + \cos \theta \sin^2 \psi) - Z_C^m \cos \delta. \end{aligned} \tag{2}$$

The motion of point D^m related to the movable system linked rigidly to the semiproduct is described by the following formulas

$$\begin{aligned} X_{ID}^m &= X_D^m \cos \frac{\psi}{Z_1} - Y_D^m \sin \frac{\psi}{Z_1}; \\ Y_{ID}^m &= X_D^m \sin \frac{\psi}{Z_1} + Y_D^m \cos \frac{\psi}{Z_1}; \\ Z_{ID}^m &= Z_D^m. \end{aligned} \tag{3}$$

The coordinates of point E^m on the sphere is calculated according to the formulas:

$$\begin{aligned} X_{IE}^m &= k_2^m Z_{IE}^m + d_2^m, \\ Y_{IE}^m &= k_1^m Z_{IE}^m - d_1^m, \\ Z_{IE}^m &= \frac{(k_1^m d_1^m - k_2^m d_2^m)}{k_1^{m2} + k_2^{m2} + 1} - \\ &= \frac{\sqrt{(k_1^m d_1^m - k_2^m d_2^m)^2 + (k_1^{m2} + k_2^{m2} + 1) \cdot (R_D^2 - d_1^{m2} - d_2^{m2})}}{k_1^{m2} + k_2^{m2} + 1}, \end{aligned} \tag{4}$$

where

$$\begin{aligned} k_1^m &= \frac{X_{ID}^m \left(X_{ID}^m \dot{X}_{ID}^m + Y_{ID}^m \dot{Y}_{ID}^m \right) + Z_{ID}^m \dot{X}_{ID}^m}{Z_{ID}^m \left(X_{ID}^m \dot{Y}_{ID}^m - Y_{ID}^m \dot{X}_{ID}^m \right)}; \\ k_2^m &= -\frac{\left(k_1^m Y_{ID}^m + Z_{ID}^m \right)}{X_{ID}^m}; \\ d_1^m &= \frac{R_D^2 \cos \beta \dot{X}_{ID}^m}{\left(X_{ID}^m \dot{Y}_{ID}^m - X_{ID}^m \dot{Y}_{ID}^m \right)}; \\ d_2^m &= \frac{\left(R_D^2 \cos \beta + d_1^m Y_{ID}^m \right)}{X_{ID}^m}. \end{aligned}$$

Point E^m projection on tooth sectional plane has the following coordinates:

$$\begin{aligned} X_E^{''m} &= \varepsilon^m \cdot X_{IE}^m, \\ Y_E^{''m} &= \varepsilon^m \cdot Y_{IE}^m, \\ Z_E^{''m} &= \varepsilon^m \cdot Z_{IE}^m, \end{aligned} \tag{5}$$

where
$$\varepsilon^m = \frac{D}{AX_{IE}^m + BY_{IE}^m + CZ_{IE}^m}.$$

The modified profile of the tooth is described by the equations

$$\xi^m = X_E^m \cos \frac{\pi}{Z_1} + [R_D \cos(\delta + \theta + \beta) + Y_E^m] \sin \frac{\pi}{Z_1};$$

$$\zeta^m = X_E^m \sin \gamma \sin \frac{\pi}{Z_1} - [R_D \cos(\delta + \theta + \beta) + Y_E^m] \sin \gamma \cos \frac{\pi}{Z_1} + [R_D \sin(\delta + \theta + \beta) + Z_E^m] \cos \gamma. \quad (6)$$

The computerised model of the planetary precessional gear on the figure 2 is presented.

Based on the analytical description of teeth profiles by a system of parametric equations on spherical surface the profilograms of the selected gearing teeth have been designed in MatchCAD. The obtained profilograms are shown in figure 3, 4, 5. The analysis of the obtained teeth profiles, based on the fundamental conditions of gearing selection (high bearing capacity due to gearing multiplicity, small dimensions and mass, technology, etc.) has allowed the selection of the teeth profiles parameters.

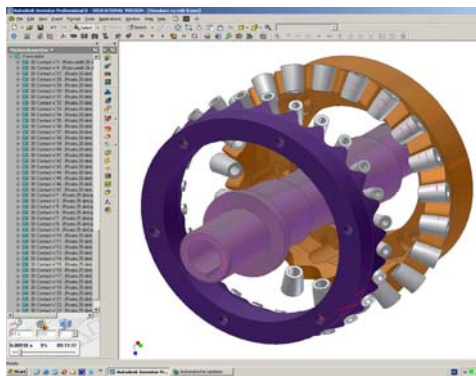


Fig. 2. Computerised model of the planetary precessional gear.

This continuing dramatical change in available computational resources offers new options in gear design for gear manufacturing processes. They include the complete 3D model of the whole gear, including the gear body and al gear flanks, obtained by using of modelling system *CATIA V5R7* (figure 6).

Constructions peculiarities and high multiplicity of gear create favourable premises for the improvement of precessional transmissions kinematics accuracy. Within these activities we elaborated:

- theoretical basis for the identification of kinematics error generated by various primary error (frontal and radial knocking), on the basis of error independent action principle by fulfilling computer assisted mathematics experiment;

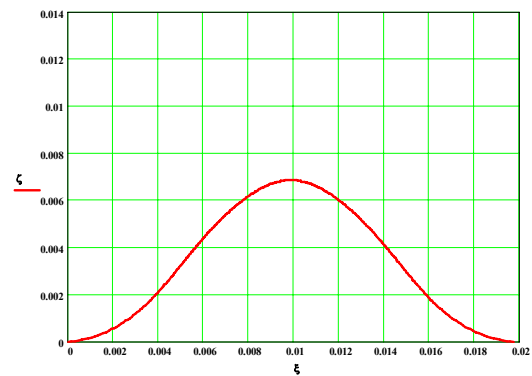
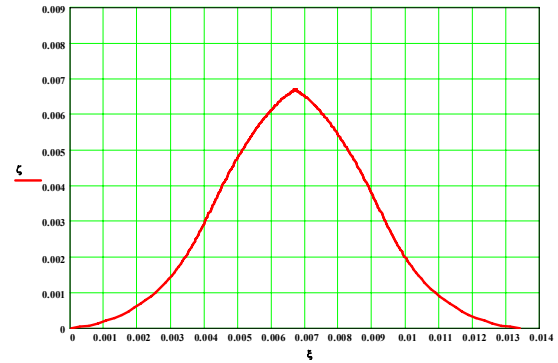


Fig. 3. The influence of the number of teeth on the form of teeth profile.

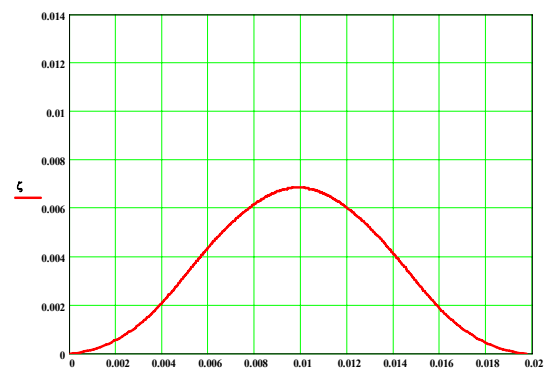
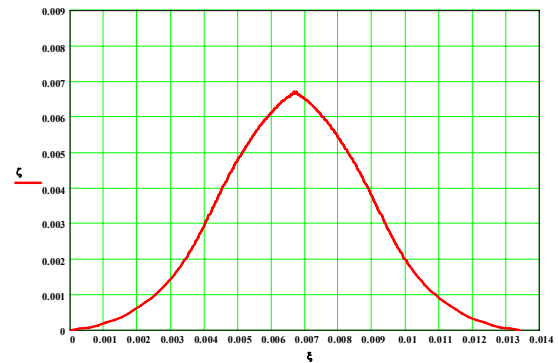


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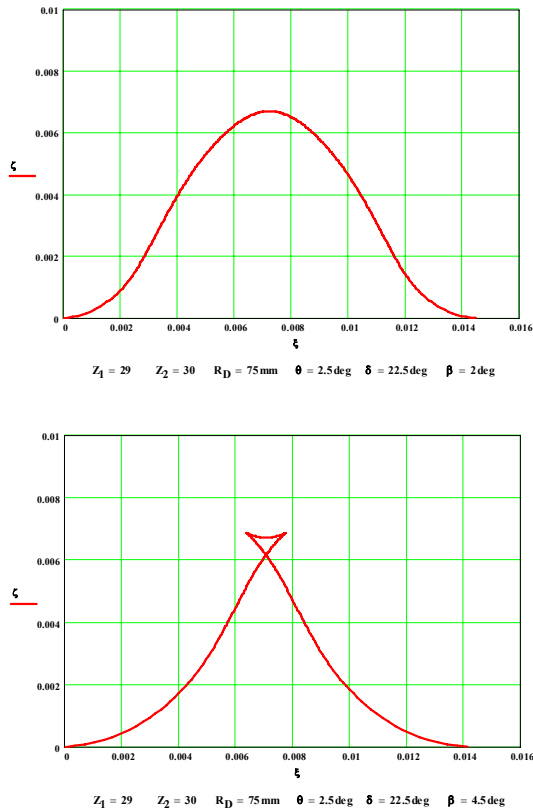


Fig. 5. The influence of the taper angle of the rollers, β on the form of teeth profile.

- compensation method for manufacturing and assembling errors;
- method of determination of probable limit error for precessional reducers with account of the stochastic character of manufacturing and assembling errors.

Special attention was paid to precessional reducers experimental research. For this purpose two laboratories were set up: 1) for mechanical tests and; 2) working technology for gear wheels. The laboratories are equipped

STUDIUL DESCRIERII ANALITICE A PROFILULUI NESTANDARD AL DINTELUI PRIN UTILIZAREA SISTEMULUI DE ECUAȚII

Rezumat. Studiul ingineresc complex al triadei „angrenaj-tehnologie –transmisie” a permis elaborarea unui nou tip de transmisie mecanică – transmisia planetară precesională cu angrenaj multipar. În această lucrare autorii prezintă modelul matematic al angrenajului multiplu. De asemenea, a fost elaborat un program de calculator pentru a face acest lucru. Se prezintă modalitățile de calcul și câteva profilograme ale dinților.

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with stands for testing and with control and modern measuring devices.

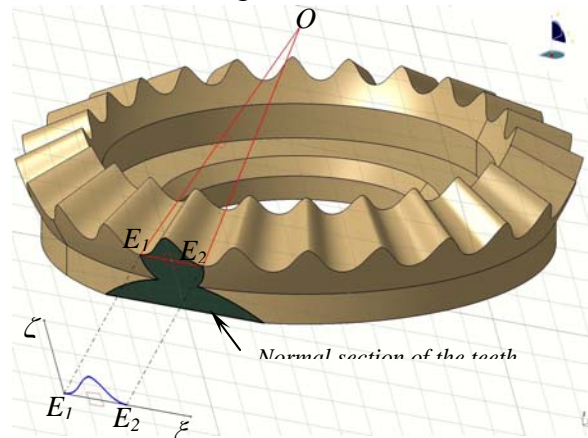


Fig. 6. 3D model of the whole gear.

Know-how in the elaboration of the multicouple precessional gear, manufacturing technology and control methods, and a range of precessional transmission diagrams belong to research team from the Technical University of Moldova. During the last 30 years the team patented about 170 inventions.

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