INVARIANTS OF ENERGY CHARACTERISTICS OF TWO-PORTS WITH VARIABLE LOADS

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Abstract

Energy characteristics as the load power and efficiency via the load resistance are the well-known two-valued cubic curves. The consideration of these quadratic fractional expressions as geometric projective transformations makes it possible to introduce the cross ratio of four points in limited single-valued working areas. The cross ratio is accepted as the regime parameter in a relative form, which is invariant to a type of the actual regime parameters and circuit sections. The form of expressions for the actual parameter changes depends on the actual regime type. Changes in regime parameters are proved; direct formulas of recalculation are proposed.

1. Introduction

Expressions for different branches of a circuit have a typical homographic or fractionally linear view for currents and resistances [1, 2], which gives solid grounds for consideration of these fractionally linear expressions as a projective transformation of projective geometry and a common use of this method [3–6].

The projective transformations preserve a cross ratio (double proportion) of four samples (values) of the variable resistance and respective currents and voltages. It is convenient to use typical regime values, which can be easily determined at a qualitative level, as the three respective samples; that is, a short circuit, an open circuit, and so on. In turn, the running regime value is the fourth sample. Therefore, the cross ratio is accepted as the regime parameter in a relative form, which is invariant to a type of the actual regime parameters and circuit sections, for example, the input–output of two-port. Hence, obvious changes in regime parameters in the form of increments are formal and do not represent the substantial aspect of the mutual influences: resistance → current.

Next, the invariant properties of two-ports allow transmitting measuring signals, using even the joint or combined wire line for communication and power supply. The cross ratio value does not depend on two-port (wire line) parameters, accuracy of measuring devices, because measuring errors of the currents mutually are reduced. In addition, the cross ratio method allows increasing the accuracy of measuring instruments with a linear-fractional scale [7, 8].

In addition to the above fractionally linear expressions, there are quadratic fractional expressions for some parameters of a circuit. For example, regulated voltage converters with limited voltage source power have two-valued regulation or stabilization characteristics as cubic curves. It was found that the cross ratio takes place in a limited unambiguous or single-valued working area of these characteristics.

In addition, some important energy characteristics of two-ports, such as the load power and efficiency via the load resistance, are similar cubic curves. In this paper, the above-mentioned results are developed for determining these energy characteristics.

2. Typical Points on Plots of Power Load Characteristics: Choice of a Single-Valued Limited Working Area

Let us consider a two-port with a variable load shown in Fig. 1.

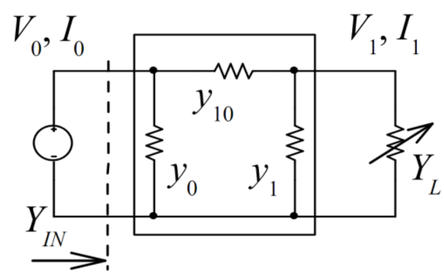

Fig. 1. Two-port with a variable load.

It is known [1, 2], that the system of equation of this two-port is as follows:

$$\begin{bmatrix} I_1 \\ I_0 \end{bmatrix} = \begin{bmatrix} -Y_{11} & Y_{10} \\ -Y_{10} & Y_{00} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_0 \end{bmatrix},$$

where Y parameters $Y_{00} = y_{10} + y_0$, $Y_{11} = y_{10} + y_1$, $Y_{10} = y_{10}$. The determinant of Y matrix $\Delta_Y = Y_{00}Y_{11} - (Y_{10})^2$.

Next, we use the equation of this two-port by the transmission a parameters

$$\begin{bmatrix} V_0 \\ I_0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \frac{1}{Y_{10}} \begin{bmatrix} Y_{11} & 1 \\ \Delta_Y & Y_{00} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}. \tag{1}$$

In turn, the admittance transformation $Y_{IN}(Y_L)$ has the following fractionally linear form:

$$Y_{IN} = \frac{I_0}{V_0} = \frac{a_{22}Y_L + a_{21}}{a_{12}Y_L + a_{11}}. \tag{2}$$

The determinant of a matrix $\Delta_A = a_{11}a_{22} - a_{12}a_{22} = 1$.

This feature of a parameters allows introducing the hyperbolic functions

$$ch^2\gamma = \frac{Y_{00}Y_{11}}{Y_{10}^2},\ sh^2\gamma = \frac{\Delta_Y}{Y_{10}^2}, \tag{3}$$

where γ is an attenuation coefficient.
Then, equation (1) can be written as

$$\begin{bmatrix} V_0 \\ \dfrac{I_0}{Y_{IN}^{CR}} \end{bmatrix} = \begin{bmatrix} ch\gamma & sh\gamma \\ sh\gamma & ch\gamma \end{bmatrix} \cdot \begin{bmatrix} V_1 \dfrac{Y_L^{CR}}{\sqrt{\Delta_Y}} \\ \dfrac{I_1}{\sqrt{\Delta_Y}} \end{bmatrix}, \tag{4}$$

where characteristic admittance at the input and output are

$$Y_{IN}^{CR} = \sqrt{\frac{Y_{00}}{Y_{11}} \Delta_Y} \, , \; Y_L^{CR} = \sqrt{\frac{Y_{11}}{Y_{00}} \Delta_Y} \, . \tag{5}$$

In addition, we introduce the normalized value

$$\overline{Y}_L = \frac{Y_L}{Y_L^{CR}} \, . \tag{6}$$

Let us use the Thévenin equivalent generator. Then, we get the open circuit OC voltage V_1^{OC} and internal conductivity Y_i as follows:

$$V_1^{OC} = \frac{Y_{10}}{Y_{11}} V_0 \, , \; Y_i = Y_{11} \, . \tag{7}$$

Next, we obtain the following voltage transfer ratio

$$K_G = \frac{V_1}{V_1^{OC}} \, , \, K_G = \frac{1}{1 + Y_L / Y_i} \, , V_1 = K_G V_1^{OC} \, . \tag{8}$$

In addition, we use an effectiveness parameter as

$$A = \frac{P_{0MAX}}{P_{GMAX}} = ch^2 \gamma \, , \tag{9}$$

where maximum powers of the voltage source and Thévenin equivalent generator at the load sort circuit are as follows:

$$P_{0MAX} = Y_{00} V_0{}^2 \, , \;\; P_{GMAX} = Y_i \cdot (V_1^{OC})^2 \, . \tag{10}$$

First, we consider a simple case of the load power via the voltage transfer ratio. Using (1), (7), and (8), we obtain

$$\frac{P_1(K_G)}{P_{GMAX}} = \tilde{P}_1(K_G) = K_G - K_G{}^2 \, . \tag{11}$$

This dependence determines a two-valued parabola with the characteristic points in Fig. 2.

We must prove the limited single-valued working area of this characteristic. To do this, we consider this parabola, as a closed curve, in the projective coordinates. Then, point S is the pole and straight line M^+M^- is the polar. Therefore, we get some symmetry or mapping of a "right-hand" working part onto "left" part. Hence, point $K_G = 1$ of the OC regime corresponds to point $K_G = 0$. In turn, points M^+, M^- are fixed or base points. This implies the correspondence of the typical and running values of $\tilde{P}_1, K_G$.

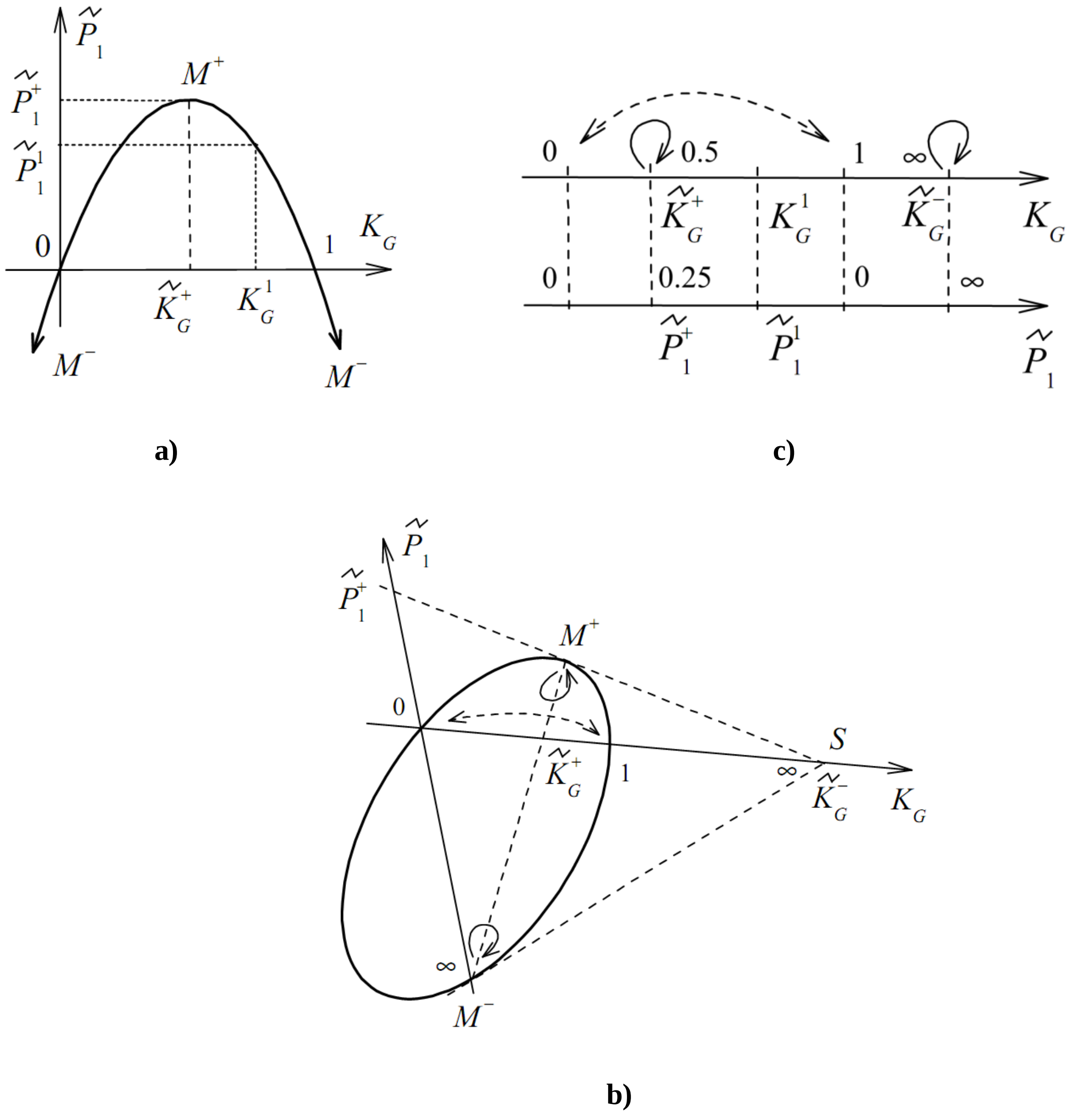

Fig. 2. Load power via the voltage transfer ratio: (a) Cartesian coordinates, (b) projective coordinates, and (c) correspondence of the typical and running values.

A more complicated case corresponds to load power P_1 via load conductivity. Using (8) and (11), this power is as follows:

$$\frac{P_1(Y_L)}{P_{GMAX}} = \frac{Y_L / Y_i}{(1 + Y_L / Y_i)^2} = \tilde{P}_1(\tilde{Y}_L) = \frac{\tilde{Y}_L}{(1 + \tilde{Y}_L)^2}. \tag{12}$$

This dependence determines a cubic curve in Fig. 4.

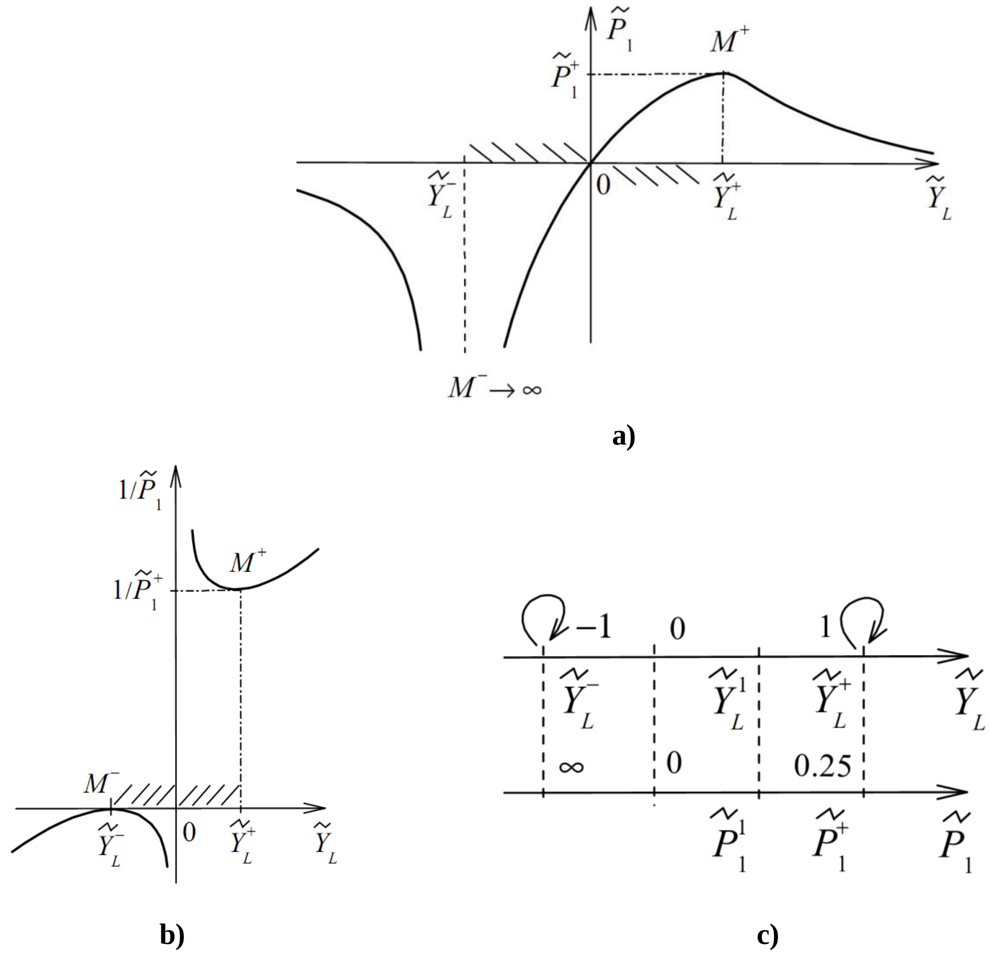

Fig. 4. (a) Load power and (b) this inverse load power via the load conductivity, and (c) correspondence of the typical and running values.

The limited single-valued working area $-1 \le \tilde{Y}_L \le 1$, which encloses OC regime, is illustrated by dash lines. For explanation of this area, we consider the inverse load power $1/\tilde{P}_1$ via load conductivity. This dependence determines the hyperbola and the correspondence of the typical and running values $\tilde{Y}_L, \tilde{P}_1$ in the single-valued working area.

The most complex case corresponds to efficiency K_P via load conductivity. Using (4)–(6), we obtain

$$K_P(\overline{Y}_L) = \frac{1}{ch^2\gamma + sh^2\gamma + ch\gamma \cdot sh\gamma \cdot \left(\overline{Y}_L + \dfrac{1}{\overline{Y}_L}\right)} . \tag{13}$$

This dependence determines a compound cubic curve in Fig. 5. Points M^+, M^- are fixed base points. In turn, $K_P = \infty$ corresponds to the two load conductivities

$$\overline{Y}_L = -\frac{1}{th\gamma} = -\frac{Y_i}{Y_L^{CR}} = -\overline{Y}_i\,,\ \overline{Y}_L = -\frac{1}{\overline{Y}_i}\,.$$

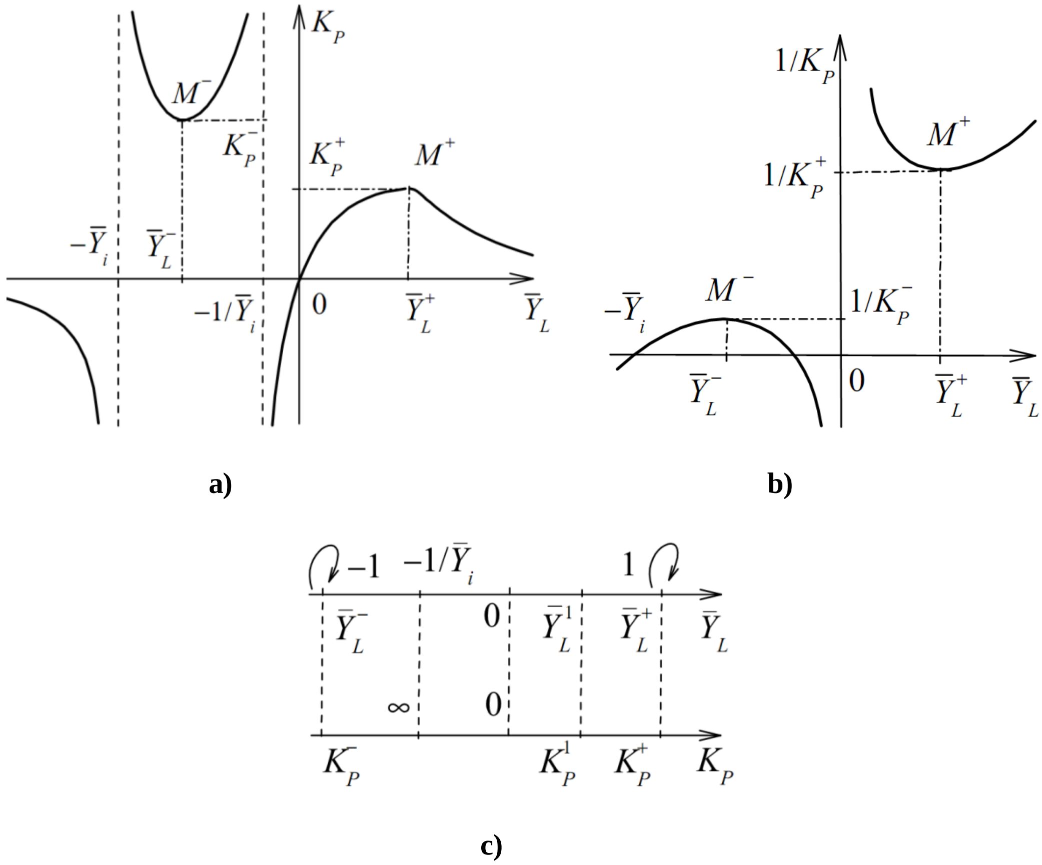

Fig. 5. (a) Efficiency and (b) this inverse efficiency via the load conductivity, and (c) correspondence of the typical and running values,

The limited single-valued working area $-1 \le \overline{Y}_L \le 1$ also encloses the OC regime. For explanation of this area, we consider inverse efficiency $1/K_P$ via load conductivity. This dependence determines the hyperbola, similar to Fig. 4b.

3. Invariant Characteristics of Load Power

We consider the load power via the load conductivity and voltage transfer ratio. For clarity, the correspondence of the typical and running values of these parameters is shown in Fig. 6.

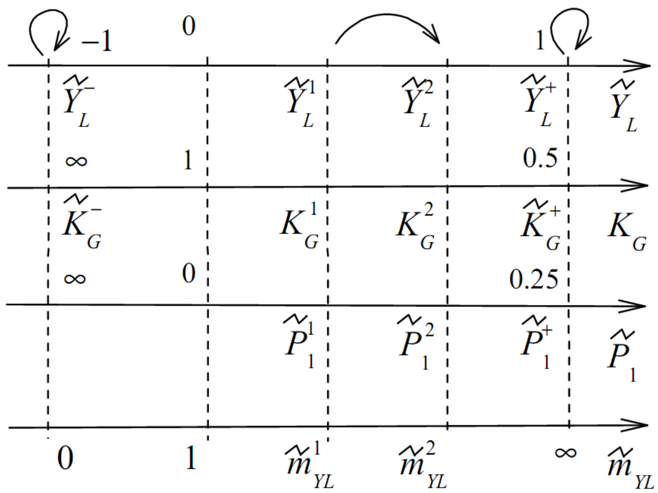

Fig. 6. Correspondence of the typical and running regime values.

The cross ratio for initial values are

$$\tilde{m}^1_{YL} = (\tilde{Y}^-_L \;\; \tilde{Y}^1_L \;\; 0 \;\; \tilde{Y}^+_L) = \frac{\tilde{Y}^1_L + 1}{\tilde{Y}^1_L - 1} \div \frac{0+1}{0-1} = \frac{1+\tilde{Y}^1_L}{1-\tilde{Y}^1_L}. \tag{14.1}$$

$$\begin{aligned} \tilde{m}^1_{KG} &= (\tilde{K}^-_G \;\; K^1_G \;\; 1 \;\; \tilde{K}^+_G) = (\infty \;\; K^1_G \;\; 1 \;\; 0.5) \\ &= \frac{K^1_G - \infty}{K^1_G - 0.5} \div \frac{1-\infty}{1-0.5} = \frac{1-0.5}{K^1_G - 0.5} = \frac{1}{2K^1_G - 1}, \end{aligned} \tag{14.2}$$

$$\tilde{m}^1_{P1} = (\infty \;\; \tilde{P}^1_1 \;\; 0 \;\; \tilde{P}^+_1) = (\infty \;\; \tilde{P}^1_1 \;\; 0 \;\; 0.25) = \frac{1}{1-4\tilde{P}^1_1}. \tag{14.3}$$

The following equality takes place

$$\tilde{m}^1_{P1} = (\tilde{m}^1_{YL})^2 = (\tilde{m}^1_{KG})^2. \tag{15}$$

Next, we consider a regime change due to a load change $\tilde{Y}^1_L \to \tilde{Y}^2_L$. Taking into account (14.1), the cross ratio for this regime changes:

$$\begin{aligned} \tilde{m}^{21}_{YL} &= \tilde{m}^2_{YL} \div \tilde{m}^1_{YL} = (\tilde{Y}^-_L \;\; \tilde{Y}^2_L \;\; \tilde{Y}^1_L \;\; \tilde{Y}^+_L) \\ &= \frac{\tilde{Y}^2_L + 1}{\tilde{Y}^2_L - 1} \div \frac{\tilde{Y}^1_L + 1}{\tilde{Y}^1_L - 1} = \frac{1 + \dfrac{\tilde{Y}^2_L - \tilde{Y}^1_L}{1 - \tilde{Y}^2_L \cdot \tilde{Y}^1_L}}{1 - \dfrac{\tilde{Y}^2_L - \tilde{Y}^1_L}{1 - \tilde{Y}^2_L \cdot \tilde{Y}^1_L}}. \end{aligned} \tag{16}$$

Then, there is strong reason to introduce a conductivity load change value as follows:

$$\tilde{Y}_L^{21} = \frac{\tilde{Y}_L^2 - \tilde{Y}_L^1}{1 - \tilde{Y}_L^2 \cdot \tilde{Y}_L^1}. \quad (17)$$

Therefore, we obtain the typical expression for the regime change:

$$\tilde{m}_{YL}^{21} = \frac{1 + \tilde{Y}_L^{21}}{1 - \tilde{Y}_L^{21}}. \quad (18)$$

Then, the subsequent value is as follows:

$$\tilde{Y}_L^2 = \frac{\tilde{Y}_L^{21} + \tilde{Y}_L^1}{1 + \tilde{Y}_L^{21} \cdot \tilde{Y}_L^1}. \quad (19)$$

Next, taking into account (14.2), we write the cross ratio or regime change for the voltage transfer ratio change $K_G^1 \to K_G^2$:

$$\tilde{m}_{KG}^{21} = (\tilde{K}_G^- \quad K_G^2 \quad K_G^1 \quad \tilde{K}_G^+) = (\infty \quad K_G^2 \quad K_G^1 \quad 0.5) = \frac{2K_G^1 - 1}{2K_G^2 - 1}. \quad (20)$$

We introduce a voltage transfer ratio change value $\tilde{K}_G^{21}$. Then, similarly to (18), the regime changes:

$$\tilde{m}_{KG}^{21} = \frac{1 + \tilde{K}_{KG}^{21}}{1 - \tilde{K}_{KG}^{21}}, \quad \tilde{K}_{KG}^{21} = \frac{\tilde{m}_{KG}^{21} - 1}{\tilde{m}_{KG}^{21} + 1}. \quad (21)$$

Using (20),

$$\tilde{K}_G^{21} = \frac{K_G^1 - K_G^2}{K_G^1 + K_G^2 - 1}. \quad (22)$$

Then, the subsequent value follows:

$$K_G^2 = \frac{K_G^1(1 - \tilde{K}_G^{21}) + \tilde{K}_G^{21}}{1 + \tilde{K}_G^{21}}. \quad (23)$$

We obtain also the group hyperbolic projective transformation with the base fixed points. For example, if the initial value is $K_G^1 = 0.5$, the subsequent value is $K_G^2 = 0.5$ for various $\tilde{K}_G^{21}$ values.

In addition, similar to (15), the following equality exists:

$$\tilde{m}_{YL}^{21} = \tilde{m}_{KG}^{21}.$$

Then, according to (21), the voltage transfer ratio change value follows:

$$\tilde{K}_{KG}^{21} = \frac{\tilde{m}_{YL}^{21} - 1}{\tilde{m}_{YL}^{21} + 1} = \tilde{Y}_L^{21}. \quad (24)$$

It is obvious that (23) has the form

$$K_G^2 = \frac{K_G^1(1-\widetilde{Y}_L^{21}) + \widetilde{Y}_L^{21}}{1+\widetilde{Y}_L^{21}}. \tag{25}$$

The obtained relationship carries out the direct recalculation of the voltage transfer ratio at a respective conductivity load change value. The main thing for practice, this group projective transformation has the base fixed points. For example, if the initial value is $K_G^1 = 0.5$, then the subsequent value is $K_G^2 = 0.5$ for various $\widetilde{Y}_L^{21}$ values.

In addition, taking into account (14.3), we obtain the cross ratio or regime change for the power change $\widetilde{P}_1^1 \to \widetilde{P}_1^2$:

$$\widetilde{m}_{P1}^{21} = (\infty \;\; \widetilde{P}_1^2 \;\; \widetilde{P}_1^2 \;\; \widetilde{P}_1^+ \;) = (\infty \;\; \widetilde{P}_1^2 \;\; \widetilde{P}_1^2 \;\; 0.25) = \frac{1-4\widetilde{P}_1^1}{1-4\widetilde{P}_1^2}. \tag{26}$$

We introduce a power change value $\widetilde{P}_1^{21}$. Then, similarly to (18), the regime changes:

$$\widetilde{m}_{P1}^{21} = \frac{1+\widetilde{P}_1^{21}}{1-\widetilde{P}_1^{21}}. \tag{27}$$

Using (26),

$$\widetilde{P}_1^{21} = \frac{\widetilde{P}_1^1 - \widetilde{P}_1^2}{\widetilde{P}_1^1 + \widetilde{P}_1^2 - 0.5}. \tag{28}$$

Then, the subsequent value follows

$$\widetilde{P}_1^2 = \frac{\widetilde{P}_1^1(1-\widetilde{P}_1^{21}) + 0.5\widetilde{P}_1^{21}}{1+\widetilde{P}_1^{21}}. \tag{29}$$

Similar to (15), the following equality takes place:

$$\widetilde{m}_{P1}^{21} = (\widetilde{m}_{YL}^{21})^2.$$

Then, using (27), we obtain

$$\widetilde{P}_1^{21} = \frac{\widetilde{m}_{P1}^{21} - 1}{\widetilde{m}_{P1}^{21} + 1} = \frac{(\widetilde{m}_{YL}^{21})^2 - 1}{(\widetilde{m}_{YL}^{21})^2 + 1}.$$

From here, the known typical expression follows:

$$\widetilde{P}_1^{21} = \frac{2\widetilde{Y}_L^{21}}{(\widetilde{Y}_L^{21})^2 + 1}. \tag{30}$$

It is obvious that (29) has the form

$$\widetilde{P}_1^2 = \frac{\widetilde{P}_1^1(1-\widetilde{Y}_L^{21})^2 + \widetilde{Y}_L^{21}}{(1+\widetilde{Y}_L^{21})^2}. \tag{31}$$

The obtained relationship performs the direct recalculation of the load power at a respective conductivity load change value. In addition, this group projective transformation has the base fixed points. For example, if the initial value is $\tilde{P}_1^1 = 0.25$, then the subsequent value is $\tilde{P}_1^2 = 0.25$ for various $\tilde{Y}_L^{21}$ values. The considered cases show that it is not correct to introduce changes of regime parameters in the form of formal increments, divisions, etc. in advance.

We now consider the efficiency via the load conductivity and voltage transfer ratio. For clarity, the correspondence of the typical and running values of these parameters is shown in Fig. 7.

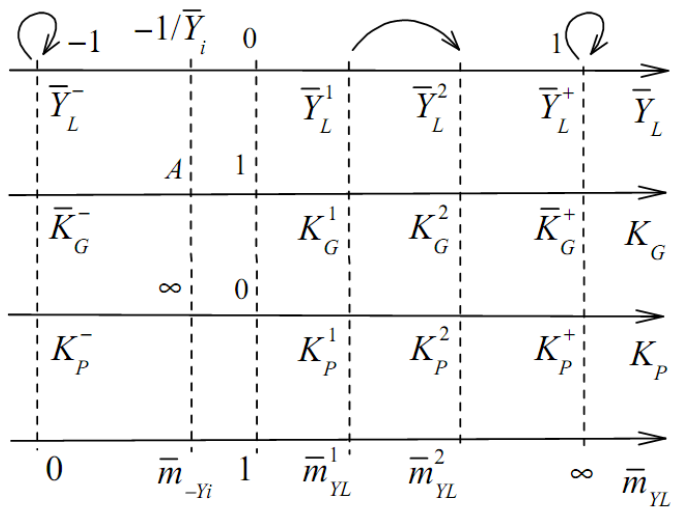

Fig. 7. Correspondence of the typical and running regime values.

The cross ratio for initial values

$$\overline{m}_{YL}^1 = (\overline{Y}_L^- \;\; \overline{Y}_L^1 \;\; 0 \;\; \overline{Y}_L^+) = \frac{1+\overline{Y}_L^1}{1-\overline{Y}_L^1}, \tag{32.1}$$

$$\overline{m}_{KG}^1 = (\overline{K}_G^- \;\; K_G^1 \;\; 1 \;\; \overline{K}_G^+) = \frac{K_G^1 - [A+\sqrt{A(A-1)}]}{K_G^1 - [A-\sqrt{A(A-1)}]} \div \frac{\sqrt{A-1}+\sqrt{A}}{\sqrt{A-1}-\sqrt{A}}. \tag{32.2}$$

$$m_{KP}^1 = (K_P^- \;\; K_P^1 \;\; 0 \;\; K_P^+) = \frac{K_P^1 - [\sqrt{A}+\sqrt{A-1}]^2}{K_G^1 - [\sqrt{A}-\sqrt{A-1}]^2} \div \frac{[\sqrt{A}+\sqrt{A-1}]^2}{[\sqrt{A}-\sqrt{A-1}]^2}. \tag{32.3}$$

The known equality

$$m_{KP}^1 = (\overline{m}_{YL}^1)^2 = (\overline{m}_{KG}^1)^2. \tag{33}$$

We consider a regime change due to a load change $\overline{Y}_L^1 \to \overline{Y}_L^2$. The cross ratio for this regime changes:

$$\overline{m}_{YL}^{21} = \overline{m}_{YL}^2 \div \overline{m}_{YL}^1 = (\overline{Y}_L^- \;\; \overline{Y}_L^2 \;\; \overline{Y}_L^1 \;\; \overline{Y}_L^+) = \frac{\overline{Y}_L^2+1}{\overline{Y}_L^2-1} \div \frac{\overline{Y}_L^1+1}{\overline{Y}_L^1-1}. \tag{34}$$

Therefore, we obtain the following request expressions at once.
The conductivity load change value

$$\overline{Y}_L^{21} = \frac{\overline{Y}_L^2 - \overline{Y}_L^1}{1 - \overline{Y}_L^2 \cdot \overline{Y}_L^1}. \tag{35}$$

The regime change

$$\overline{m}_{YL}^{21} = \frac{1 + \overline{Y}_L^{21}}{1 - \overline{Y}_L^{21}}. \tag{36}$$

The subsequent value corresponds to hyperbolic projective transformation (19):

$$\overline{Y}_L^2 = \frac{\overline{Y}_L^{21} + \overline{Y}_L^1}{1 + \overline{Y}_L^{21} \cdot \overline{Y}_L^1}. \tag{37}$$

Next, we write the cross ratio for the voltage transfer ratio change $K_G^1 \to K_G^2$:

$$\overline{m}_{KG}^{21} = (\overline{K}_G^- \quad K_G^2 \quad K_G^1 \quad \overline{K}_G^+) = \frac{K_G^2 - [A + \sqrt{A(A-1)}]}{K_G^2 - [A - \sqrt{A(A-1)}]} \div \frac{K_G^1 - [A + \sqrt{A(A-1)}]}{K_G^1 - [A - \sqrt{A(A-1)}]}. \tag{38}$$

Similarly, we introduce a voltage transfer ratio change value $\overline{K}_G^{21}$. Then, similar to (21), the regime change

$$\overline{m}_{KG}^{21} = \frac{1 + \overline{K}_{KG}^{21}}{1 - \overline{K}_{KG}^{21}}. \tag{39}$$

Using (38), the voltage transfer ratio change value

$$\overline{K}_G^{21} = \sqrt{A(A-1)} \frac{K_G^2 - K_G^1}{K_G^2 \cdot K_G^1 - A(K_G^2 + K_G^1 - 1)}. \tag{40}$$

Then, the subsequent value

$$K_G^2 = \frac{AK_G^{21}(K_G^1 - 1) - K_G^1 \sqrt{A(A-1)}}{K_G^{21}(K_G^1 - A) - \sqrt{A(A-1)}}. \tag{41}$$

The known equality

$$\overline{m}_{YL}^{21} = \overline{m}_{KG}^{21}.$$

Then, according to (39), the voltage transfer ratio change value follows:

$$\overline{K}_{KG}^{21} = \frac{\overline{m}_{YL}^{21} - 1}{\overline{m}_{YL}^{21} + 1} = \overline{Y}_L^{21}. \tag{42}$$

It is obvious that (41) gets the form

$$K_G^2 = \frac{K_G^1[A\overline{Y}_L^{21} - \sqrt{A(A-1)}] - A\overline{Y}_L^{21}}{K_G^1\overline{Y}_L^{21} - [A\overline{Y}_L^{21} + \sqrt{A(A-1)}]}. \quad (43)$$

The obtained relationship carries out the direct recalculation of the voltage transfer ratio at a respective conductivity load change value. In addition, this group projective transformation has the base fixed points. For example, if the initial value is $K_G^1 = A \pm \sqrt{A(A-1)}$, then the subsequent value is $K_G^2 = A \pm \sqrt{A(A-1)}$ for various values $\overline{Y}_L^{21}$.

As the last step, we get the cross ratio for the efficiency change $K_P^1 \to K_P^2$:

$$\overline{m}_{KP}^{21} = (K_P^- \;\; K_P^2 \;\; K_P^1 \;\; K_P^+) = \frac{K_P^2 - [\sqrt{A} + \sqrt{A-1}]^2}{K_G^2 - [\sqrt{A} - \sqrt{A-1}]^2} \div \frac{K_P^1 - [\sqrt{A} + \sqrt{A-1}]^2}{K_G^1 - [\sqrt{A} - \sqrt{A-1}]^2}. \quad (44)$$

We introduce an efficiency change value $\overline{K}_P^{21}$. Then, the regime change

$$\overline{m}_{KP}^{21} = \frac{1 + \overline{K}_P^{21}}{1 - \overline{K}_P^{21}}. \quad (45)$$

Using (44), the efficiency change value

$$\overline{K}_P^{21} = 2\sqrt{A(A-1)}\frac{K_P^2 - K_P^1}{K_P^2 \cdot K_P^1 - (2A-1)\cdot(K_P^2 + K_P^1) + 1}. \quad (46)$$

Then, the subsequent value

$$K_P^2 = \frac{\overline{K}_P^{21}[(2A-1)K_P^1 - 1] - 2\sqrt{A(A-1)}\,K_P^1}{\overline{K}_P^{21}[K_P^1 - (2A-1)] - 2\sqrt{A(A-1)}}. \quad (47)$$

The known equality

$$\overline{m}_{KP}^{21} = (\overline{m}_{YL}^{21})^2.$$

Therefore, using (45),

$$\overline{K}_P^{21} = \frac{\overline{m}_{KP}^{21} - 1}{\overline{m}_{KP}^{21} + 1} = \frac{(\overline{m}_{YL}^{21})^2 - 1}{(\overline{m}_{YL}^{21})^2 + 1}.$$

From here, the known typical expression follows:

$$\overline{K}_P^{21} = \frac{2\overline{Y}_L^{21}}{(\overline{Y}_L^{21})^2 + 1}.$$

It is obvious that (47) has the form

$$K_P^2 = \frac{K_P^1\{2\overline{Y}_L^{21}(2A-1) - 2\sqrt{A(A-1)}[(\overline{Y}_L^{21})^2 + 1]\} + 2\overline{Y}_L^{21}}{2K_P^1\overline{Y}_L^{21} - \{2\overline{Y}_L^{21}(2A-1) + 2\sqrt{A(A-1)}[(\overline{Y}_L^{21})^2 + 1]\}}.$$

The obtained relationship performs the direct recalculation of the efficiency at a respective conductivity load change value. Also, this group projective transformation has the base fixed points. For example, if the initial value is $K_P^1 = (\sqrt{A} \pm \sqrt{A-1})^2$, then the subsequent value is $K_P^2 = (\sqrt{A} \pm \sqrt{A-1})^2$ for various values $\overline{Y}_L^{21}$.

Thus, this well-founded introduction of regime parameter changes shows that the form of expressions for the subsequent values depends on the type of the regime. Therefore, it will be not correct to introduce formal increments, divisions, etc.

4. Conclusions

(i) The cross ratio for quadratic fractional expressions is carried out in a limited single-valued working area.
(ii) The cross ratio is accepted as the regime parameter in a relative form, which is invariant to the type of the actual regime parameters and circuit sections and depends on the type of the actual regime.
(iii) Changes in regime parameters are proved; direct formulas of recalculation are proposed.
(iv) The application of this approach to the alternating current circuits is a promising direction of researches.
(v) The represented invariant properties of energy characteristics give the base for the determination of single-valued areas of various cubic expressions.

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