DETERMINATION OF DEVIATION FROM THE MAXIMUM POWER REGIME OF A PHOTOVOLTAIC MODULE

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The problem of the determination of the photovoltaic module (converter) regime in the normalized or relative form is examined. The parameters of characteristic regimes, such as the maximum power regime, short circuit, open circuit, are considered as the base values. Utilized linear- hyperbolic approximation of the current-voltage characteristic gives the interpretation of the deviation of regime from the maximum power regime as the rotation of a radius-vector in the projective coordinate system. The obtained relationships make it possible to compare the effectiveness of the current regimes of photovoltaic converters with the different parameters.

1. The problem of implementation of the effectiveness of photovoltaic converters

In photovoltaic power systems the maximum power point trackers (MPPTs) are used extensively [1]. Usually, a solar array (SA) is a set of in parallel and series-connected modules. Then, it is possible to use either a common or individual MPPT for each module.

In the case of common MPPT, the differences in the parameters of the modules will appear. This leads to the deviation of the regime of modules within large range from its maximum power point, although, as a whole, a battery works under these conditions. If individual MPPTs are used, then these devices can be turned on only in the case the sufficiently large deviations of regime. Therefore, it is necessary to correctly determine these deviations over a wide range of load change for each module, which will make it possible to estimate the load of modules, to establish equivalence and effectiveness of their regimes. This problem is characteristic for the entire class of the related tasks in the theory of the electrical circuits, power electronics, and power engineering.

2. Determination of the parameters of regime in the relative form

In addition to the concrete parameters of regimes, such as current, voltage, and power loading, it is necessary to determine regime in the normalized or relative form for the estimation of effectiveness. This makes it possible to be abstracted from the concrete parameters of the regime, parameters of the energy source, to use similarity principle [2], to determine the effectiveness of the regime in the of form assigned criterion [3]. Relative expressions on the prevailing practice are comprised with the aid of the characteristic value (as scale) of the corresponding parameter of regime. In the case of several characteristic values, the problem of the selection of scale and composition of quite relative expressions has already been manifested. Also, changes in the parameters of regime are examined, which shows a change in the effectiveness of the energy source [4]. Changes are introduced either through the ratio of the initial and subsequent values of the regime parameters or through their difference in reference to the parameter values proper (in “the percentages”). Thus, for the parameters or indices of a regime, there are two possibilities: either the initial composition of relative expressions or the composition of the analytical expressions through the already comprised or base expressions for other indices of the regime. In the general case, different dependences for one and the same index are obtained, which introduces uncertainty in their selection. Thus, the absence of the systematically substantiated approach or axiomatic is manifested for the composition of similar relative expressions.

Let us demonstrate the given reasoning about the determination of the regime parameters and their changes in the relative form taking into account the special features of current-voltage (CV) characteristic of a solar array (Fig. 1, curve *1*).

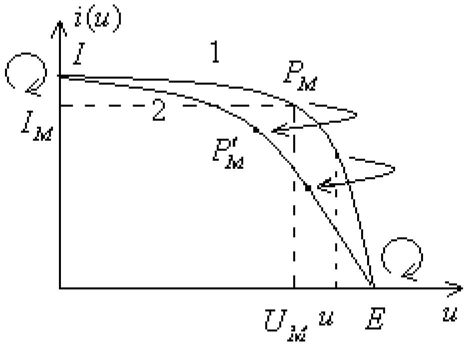


Fig. 1. Standard CV characteristics of two SA with the different parameters.

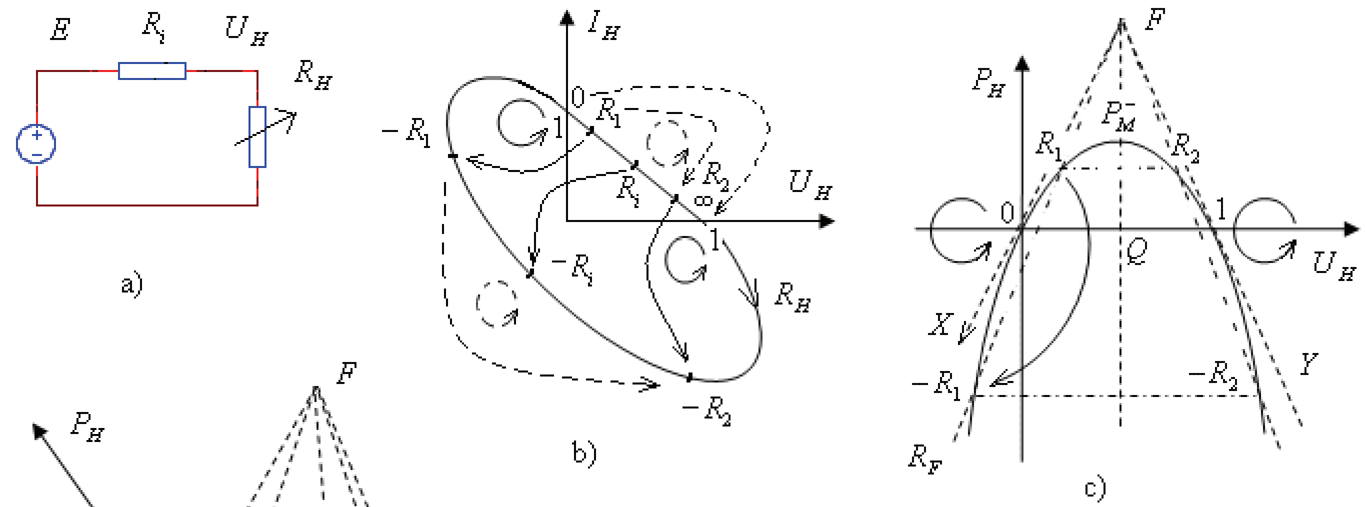
The CV characteristic is nonlinear and takes the typical convex form with the maximum power point P_M. In addition to such characteristic values as current I_M and voltage U_M, there are also the open-circuit (OC) voltage E and short-circuit (SC) current I. We shall compare the regimes of the different SAs, in which the coordinates of the maximum power points or fill factors are distinguished. The pointers show the obvious correspondence of the points characteristic regimes. It is necessary to determine the correspondence of current or intermediate regimes.

3. Determination of the relative regimes of the electrical circuits

On the basis of projective geometry [5, 6] in the cycle of papers [7-10], the method of the analysis of electrical circuits with the variable regimes is developed. Let us give the necessary information from the theory of this method using the example of simple circuit with the linear characteristic. Thus, a deeper analysis will serve a systematic basis for investigating the nonlinear CV characteristic. Figure 2 represents the CV straight line $I_H\,(U_H)$ and the parabola of power $P_H\,(U_H)$ in accordance with the normalized expressions:

$$I_H / I = 1 - U_H / E\,,\ P_H = (U_H / E) - (U_H / E)^2\,, \qquad (1)$$

where $I = E / R_i$ is the SC current. In the first quadrant, the voltage source E returns the energy into the load, therefore there is the maximum power point P_M^-, when $R_H = R_i$. At the SC and OC points, when $R_H = 0, \infty$, the power is equal to zero. So, the calibration of the linear CV characteristic in the values of resistance is carried out. The load can also return energy into the voltage source E; in this case, the load voltage is $U_H < 0$ and is $U_H > 1$, and the load resistance is $R_H < 0$ for these two regions. Therefore, it is possible to continue calibration to the left from the



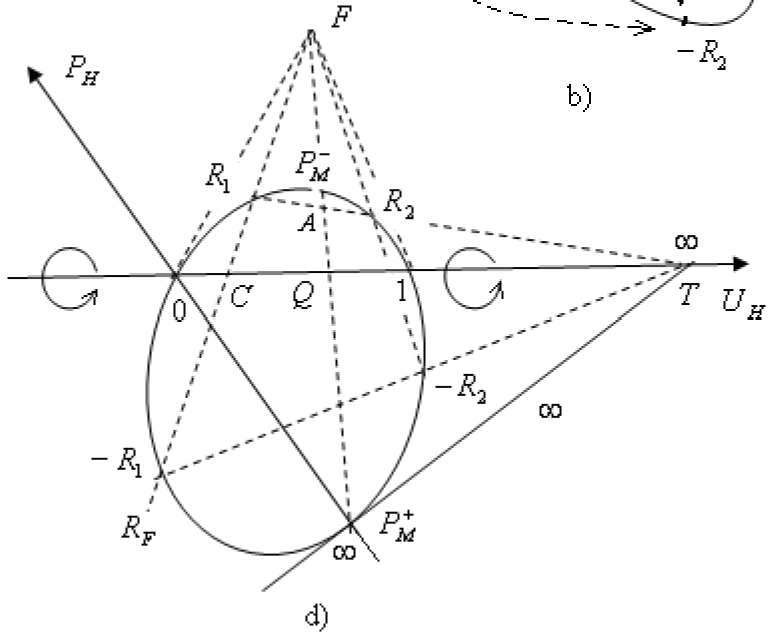


Fig. 2. Characteristics of the simple circuit: (a) circuit diagram, (b) linear CV characteristic, parabola of power in (c) Cartesian and (d) projective coordinate system.

point $R_H = 0$ and to the right from the point $R_H = \infty$. In this case, all the values of output power will all increase. In the final analysis, for the resistance $R_H = -R_i$ power is $P_M^+ = \infty$, therefore, the calibrations of CV characteristic will coincide precisely at the point $-R_i$, i.e., straight line in the projective geometry is closed at the infinitely remote point. Also, the parabola is a closed oval curve, which concerns the infinitely remote straight line TP_M^+ at the point P_M^+.

Let us consider four points of the characteristic regimes, which correspond to $+R_i, -R_i, 0, \infty$.

In the projective geometry, three points of straight line are independent variables, and any fourth is expressed as these points. Cross-ratio serves as the characteristic of the mutual arrangement of four points. For the arbitrary point, the cross-ratio m assigns the projective coordinate of point on the straight line or, in our understanding, regime in the relative form:

$$m = m(R) = (0 \;\; R \;\; R_i \;\; \infty) = \frac{R-0}{R-\infty} : \frac{R_i - 0}{R_i - \infty} = \frac{R}{R_i}. \tag{2}$$

For the SC and OC points $m = 0, m = \infty$. Cross-ratio is expressed invariantly via remaining variables U_H, I_H taking into account the correspondence of the points:

$$m = m(U_H) = (0 \;\; U \;\; 0{,}5 \;\; 1) = \frac{U}{1-U} = m(I_H) = (1 \;\; I \;\; 0{,}5 \;\; 0) = \frac{1-I}{I}. \tag{3}$$

The special case of the fourth point of the type $-R_i$ is the property of the harmonic conjugacy of four points, which determines the symmetry of points $-R_i$, R_i relative to base or extreme 0, ∞.

In this case, $m(-R_i) = -1$. Physically this symmetry corresponds to mapping the region of power consumption by load on the region of return. For this, the mapping of the points of the parabola of region $P>0$ onto the region $P<0$ is realized from the point F. In this case, points 0, 1 of axis U are fixed, and the points of parabola $R_i, -R_i$ pass into each other. The point F is formed due to the intersection of the tangential FX, FY at fixed points. This point is called pole, and the straight line, that passes through fixed points 0, 1 is the polar. The indicated symmetry is obtained relative to polar.

Since the extreme points of the regime change are points SC and OC, then we come to the projective system of coordinates YFX. In this coordinate system the polar is considered as the infinitely remote straight line. Therefore, the initial parabola is already hyperbola, and the axis of the coordinates FX, FY are asymptotes. In this case, the point on the hyperbola is assigned as the rotation of a radius-vector $R_F F$ from the initial position at point P_M^-.

Non-Euclidean distance $R_1 P_M^-$ is determined by the hyperbolic arc length of hyperbola. Also, the symmetry of points R_1, R_2 relative to point R_i as the points of equal power is manifested. The mapping of the points R_1, R_2 relative to straight P_M^-, P_M^+ of parabola leads to the additional system of pole and polar, point T is a pole, and straight line $P_M^-\ P_M^+$ is a polar. Thus, we obtained the coordinated picture or the "kinematics" diagram of the deviation of regimes relative to the selected initial point and extreme points.

4. Proposed determination of the deviation of the regimes of solar array

In order to use the examined approach, it is necessary to obtain the approximation of the CV characteristic of the SA in the form of the second order curve. This model is developed [11-13] on the basis of similarity with the characteristic of active two-terminal circuit with the self-restriction of the current [14]:

$$u(i) = u_1(i) + u_2(i) = E_1 A \frac{1 - i/I}{A - i/I} + E_2 (1 - i/I). \qquad (4)$$

The parameter A in formula (4) determines the degree of the convexity of hyperbolic component $u_1(i)$. The axes YS, XS are the asymptotes of hyperbola $u(i)$ as shown in Fig. 3a.

For the demonstration of this approximation we utilize the set of characteristics for the different level of the illumination of an industrial solar array MSX120 [15]. According to the actual data for two points near the point P_M, we calculate the approximation parameters represented in the table.

Table.

Illumin.	I, A	E, V	E_1	E_1/E	A
1	3.8103	21.054	18.054	0.8575	1.0035
0.8	3.048	20.783	17.89	0.86	1.00382
0.6	2.2896	20.48	17.71	0.864	1.00371
0.4	1.531	20	17.74	0.887	1.00578

Data analysis shows that the parameters E_1/E and A approximately preserve their values for the different illumination levels. This is connected with constancy of proportions of the I_M/I I and U_M/E. The calculated family of curves $i(u)$ is given in Fig. 3b.

Now let us examine the constructions in Fig. 4a for determining the deviation from the point P_M. For this, we will use the results of calculations [14], where the CV characteristic of the quasi-resonance voltage converter is examined. We utilize, for the sake of clarity, an example of the dependence $i(u)$ (table) for the complete illumination level. Coordinates of point P_M are: $U_M = 0.8069, I_M = 0.9535$.

The point F is a pole, and straight TQ is a polar. In this case, the mapping of region $P > 0$ onto the region $P < 0$ occurs.

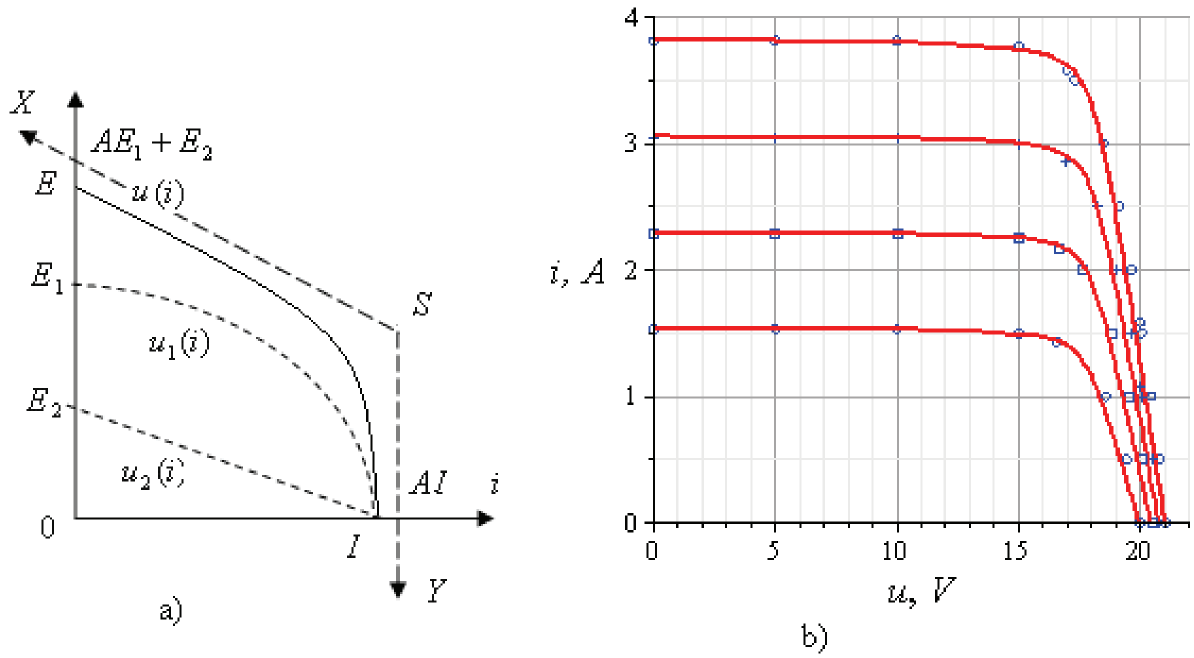


Fig. 3. Approximation of the CV characteristic in the form of the second order curve: (a) hyperbola $u(i)$, (b) the family of curves of approximation (solid lines) and actual values (point).

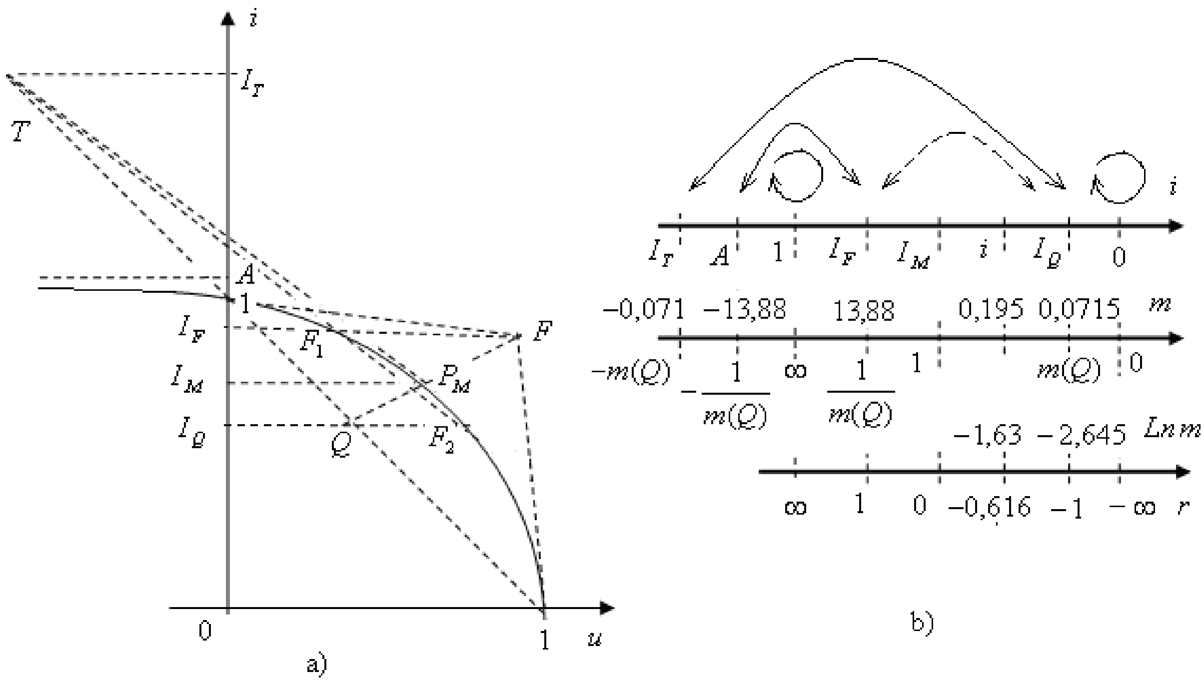


Fig. 4. Points of the characteristic regimes: (a) arrangement on CV characteristic proper, (b) the interrelation of the points between themselves.

By analogy, the point T is a pole, the straight FQ is a polar; in this case, the symmetry of curve relative to point P_M occurs. Coordinates of the points F, T, Q are:

$$I_F = \frac{A}{2A-1} = 0.996524,\ I_T = 3.1455,\ I_Q = 0.59443.$$

Let us isolate all points of characteristic regimes in the CV characteristic. These are the known points for the maximum power P_M with the current I_M, for SC regime $i=1$ and OC regime $i=0$. Also, new points are shown, which correspond to the limiting values of the current $i=A$, points F_1 and F_2 with the currents I_F, I_Q and current I_T. Further, it is necessary to reveal which points are used as a base and which points depend on this base. For this, we depict the correspondence of all points in Fig. 4, and assume points 0, 1 as base or extreme. The mutual mapping of points I_T, I_Q relative to points 0,1 is shown by pointers. Points 0, 1 and I_T, I_Q are harmonic conjugacy; therefore, point I_T depends on point I_Q. Also, points 0, 1 and I_F, A are harmonic conjugacy; therefore, the point A depends on the point I_F. We assume the point I_M as the initial one. If we verify the values of cross-ratios for each of the points I_F, I_Q relative to initial point I_M and base 0,1, then these values are mutually-reverse because of the above-indicated symmetry of points F_1, F_2. For the sake of clarity, the mapping of the points I_F, I_Q relative to initial point I_M is shown by dotted line. By analogy, the value of cross-ratios for the remaining characteristic points I_T, A relative to the initial point I_M and base 0,1 is mutually-reverse. Thus, all characteristic points are reduced to one system and are evinced through the scale point, which it is shown for the axis m. We assume the cross-ratio $m(Q)$ as the scale, then any point i of the current regime (e.g., $i=0.8$) can be expressed through the cross-ratio relative to reference points and initial point:

$$m(i) = (0\ i\ I_M\ 1) = \frac{i}{i-1} : \frac{I_M}{I_M-1} = -\frac{0.8}{0.8-1} : 20.5 = 0.195. \qquad (5)$$

It is convenient to introduce hyperbolic distance $S(m) = Ln\, m$, then for the current observed value $S(i) = Ln\, m(i) = -1.63$. By analogy, the scale value of the distance is $S(Q) = Ln\, m(Q) = 2.645$, then the normalized distance from the point of the maximum power is $r = S(i)/S(Q) = -0.616$. If the deviation r is assigned, we find an actual value $i(m)$ taking into account $m(Q)$. Further, it is convenient to introduce the step of the deviation Δr, then the deviation $r = k \cdot \Delta r$, where k is a number of the step of a sequential change in the regime. A calculation according to $i(m)$ gives the value of current $i(k)$ at each step. If we carry out similar calculations for the compared SA, the calibration of CV characteristics is obtained. Let us examine the special case, when the point $T \to \infty$. For this, we utilize the straight line FS, which passes through the point $u = E_2$ in Fig. 5a.

This straight line is the axis of the usual symmetry of hyperbola in the coordinate system $Y\,S\,X$, therefore, $I_Q = 0.5$. The coordinate of point P_{E2} is: $I_{E2} = A - \sqrt{A(A-1)} = 0.9442$, which is close to the actual value of the current of maximum power, in consequence of which it is possible to accept the point I_{E2} for the initial. For point Q: $m(Q) = (0\ I_Q\ I_{E2}\ 1) = \sqrt{(A-1)/A} = 0.059$, and expression for calculating the current according to the deviation takes the form: $i = m^{r-1}(Q)/(1+m^{r-1}(Q))$. For this simplified model, the parameters A, E_1 are easily defined through the parameters of the point P_M of actual curve:

$$A = \frac{I_M{}^2}{2I_M - 1} = 1.0042, \; E_1 = \frac{U_M - 1 + I_M}{2I_M - 1} = 0.85. \quad (6)$$

For example, the deviations for the family of CV characteristics of solar array MSX120 are calculated with the different illumination levels, which is shown in Fig. 5b. For all the curves, it is accepted $A = 1.004$, $\Delta r = 0.3$, $k = 0,1,..5$.

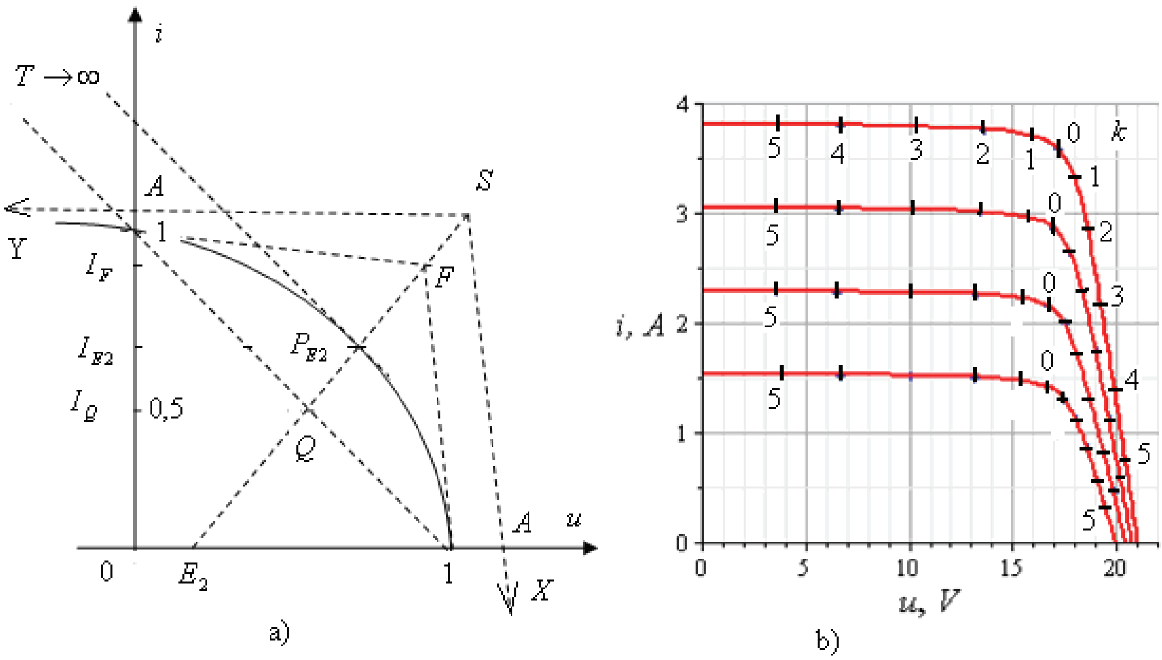


Fig. 5. A special case of the characteristic points: (a) arrangement on CV characteristic proper (b) example of the calibration against the deviation of family of CV characteristics.

5. Conclusions

The formulated problem shows its complexity and requires studying and using entire totality of the points of characteristic regimes. The developed approach uses a convenient geometric apparatus, which makes it possible to soundly introduce a number of ideas for the development of the special features of CV characteristics. The obtained results make it possible to analyze the effectiveness of the regimes both of SA operating in one electric power supply system and the SA in the different systems. In the systematic plan, it is useful to examine the place of the proposed approach in different fields of science; this is the use of invariants and symmetry. First of all, these are the mechanics of relative motion (recalculation of the coordinates of point in different reference frames) and biology (laws of age-related changes in the organisms) [16], which makes it possible to formulate the concept of the generalized description of the behavior of different systems.

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