On a Certain Property of the Elements of a Finitely Generated Lattice

Ion Negru

Technical University of Moldova, Chişinău, Republic of Moldova
e-mail: ion.negru1941@gmail.com

Firstly, a reminder that a lattice is a set $\boldsymbol{S}$ of elements; it is partially ordered, closed in relation to two lattice operations: the reunion $\boldsymbol{a+b}$ and the intersection $\boldsymbol{a\cdot b}$ of any two elements $\boldsymbol{a}$ and $\boldsymbol{b}$ from set $\boldsymbol{S}$. (The reunion $\boldsymbol{a+b}$ is the smallest element of the lattice containing both elements $\boldsymbol{a}$ and $\boldsymbol{b}$; the intersection $\boldsymbol{a\cdot b}$ is the greatest element of the lattice contained in both elements $\boldsymbol{a}$ and $\boldsymbol{b}$. Obviously, $\boldsymbol{a\leq a+b, b\leq a+b, a\geq a\cdot b, b\geq a\cdot b}$)
A lattice may also be defined thusly: the generating elements of the lattice are given. Other elements, different from the generators, are obtained via the two lattice operations, applied to the generators.

Statement. Let $\boldsymbol{a1, a2, a3, \cdots}$,, an be the lattice generators. And let $\boldsymbol{T}$ be any element (term) of the lattice. The following takes place:

$$\boldsymbol{T \geq a_1 + a_2 + \cdots + a_{i-1} + a_{i+1} + \cdots + a_n}$$

or:

$$\boldsymbol{T \geq a_i, (i = 1, 2, 3, \cdots, n)}.$$

The statement is proven through the method of mathematical induction in relation to the length of element (term) $\boldsymbol{T}$. (Elements of length 1 are, evidently, the lattice generators. Any other element $\boldsymbol{T}$ of a length greater than 1 is presented as $\boldsymbol{T = T_1 + T_2}$ or $\boldsymbol{T = T_1 \cdot T_2}$, where the lengths of $\boldsymbol{T_1}$ and $\boldsymbol{T_2}$ are less than the length of element $\boldsymbol{T}$ - this presentation is deployed within the application of the method of mathematical induction.)